Name:____________________________________ Quiz Score:_____/10

Answer each question completely in the area below. Show all work and explain your reasoning. If the work is at all ambiguous, it is considered incorrect. No phones, calculators, or notes are allowed. Anyone found violating these rules will be asked to leave immediately. Point values are in the square to the left of the question. If there are any other issues, please ask the instructor.

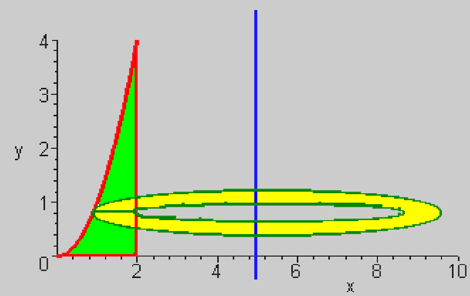
1. **Using the method of washers**, we will compute the volume of the solid obtained by the region enclosed by:

$$y = x^2, \qquad x = 0, \qquad x = 2,$$

around the vertical line $x = 5$.

4 (a) First, find the area of a single slice. *Hint:* draw the picture. Decide which direction the slice is in and then identify the inner and outer radii.

Solution:



The picture is above. Notice, for the washer method, we slice **perpendicular** to the line we are rotating around, meaning we must take slices in the y-direction.

From the picture, it's clear the inner radius is defined by the distance from the line $x = 5$ to the line $x = 2$, which means $r_{\text{in}} = 3$. The outer radius is defined by the distance from $x = 5$ to our outer function $y = x^2$, which suggests that $\sqrt{y} = x$. Thus, $r_{\text{out}} = 5 - \sqrt{y}$.

We know, for the washer method:

$$A(y) = \pi\left[r_{\text{out}}^2 - r_{\text{in}}^2\right] = \pi\left[(5-\sqrt{y})^2 - 3^2\right].$$

2 (b) Compute the total volume of the shape.

Solution: Now that we have the area of a slice, we know the volume is obtained by adding up all of the slices via an integral:

$$V = \int_0^4 A(y)\,\mathrm{d}y.$$

Notice, we are integrating from 0 to 4 because the intersections occur at $x = 0, y = 0$ and

$x = 2, y = 4$. Computing this integral yeids:

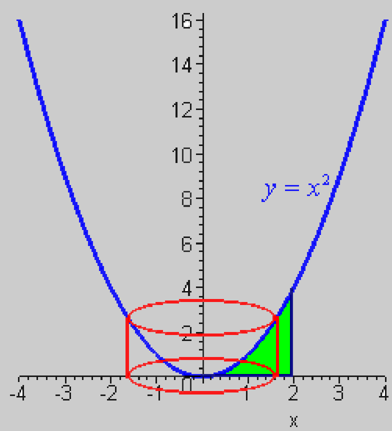
$$\begin{aligned} V = \int_0^4 &= \pi \left[(5 - \sqrt{y})^2 - 3^2 \right] \, dy \\ &= \pi \int_0^4 [25 - 10\sqrt{y} + y] \, dy \\ &= \pi \left[25y - 10\frac{y^{3/2}}{3/2} + \frac{y^2}{2} \right]_{y=0}^{y=4} \\ &= \pi \frac{56}{3}. \end{aligned}$$

4 2. **Using the method of cylindrical shells**, compute the volume of the solid obtained by rotating the region enclosed by

$$y = x^2, \qquad x = 0, \qquad x = 2,$$

around the y-axis.

Solution:



This problem is slighly easier and therefore worth fewer points. For shells, we take slices **parallel** to our axis of rotation, meaning we take a slice in the x-direction. From there, we compute the volume of a single shell:

$$V_{\text{shell}} = \text{circumference} \cdot \text{height} \cdot \text{thickness}$$

Here, we see from the picture that the radius of an arbitrary shell is $r = x$, meaning the cirumference is $C = 2\pi r = 2\pi x$. The height is defined by the height of the function, meaning $h = x^2$. The thickness of the shell would be our thin slice length, $\Delta x \to dx$. Thus:

$$V = \underbrace{2\pi x}_{\text{cirumference}} \underbrace{x^2}_{\text{height}} \underbrace{dx.}_{\text{thickness}}$$

Adding up all the shells, equivalent to integrating over different x values, yields the total volume:

$$\begin{aligned} V &= \int_0^2 (2\pi x)x^2 \, \mathrm{d}x \\ &= \int_0^2 2\pi x^3 \, \mathrm{d}x \\ &= 2\pi \int_0^2 x^3 \, \mathrm{d}x \\ &= 2\pi \frac{x^4}{4}\bigg|_{x=0}^{x=2} \\ &= 8\pi. \end{aligned}$$