(1) Prove the following properties of the matrix $A$ formed in the finite difference methods for Poisson equation with Dirichlet boundary condition:
   (a) it is symmetric: $a_{ij} = a_{ji}$;
   (b) it is diagonally dominant: $a_{ii} \geq \sum_{j=1,j\neq i}^{N} a_{ij}$;
   (c) it is positive definite: $u^TAu \geq 0$ for any $u \in \mathbb{R}^N$ and $u^TAu = 0$ if and only if $u = 0$.

(2) Let us consider the finite difference discretization of Poisson equation with Neumann boundary condition.
   (a) Write out the $9 \times 9$ matrix $A$ for $h = 1/2$.
   (b) Prove that in general the matrix corresponding to Neumann boundary condition is only semi-positive definite.
   (c) Show that the kernel of $A$ consists of constant vectors: $Au = 0$ if and only if $u = c$.

(3) Implement finite difference methods for solving the Poisson equation $u'' = f$ with pure Dirichlet or pure Neumann boundary in the unit interval $(0, 1)$.
   (a) Discretize the domain into uniform grid.
   (b) Discretize the $\Delta$ operator for interior points.
   (c) Modify the equation for boundary points.
   (d) Use direct solver in Matlab to compute the solution: $u = A\backslash f$.
   (e) Choose a smooth function and substitute into Poisson equation to get a right hand function. Then compare your computation with the true solution in the maximal norm, i.e., compute $\|u_I - u_h\|_{\infty}$. 