HOMEWORK 1 OF MATH 226 A, FALL 2009

(1) Prove the following properties of the matrix A formed in the finite difference methods for Poisson equation with Dirichlet boundary condition:
  (a) it is symmetric: $a_{ij} = a_{ji}$;
  (b) it is diagonally dominant: $a_{ii} \geq \sum_{j=1, j \neq i}^{N} a_{ij}$;
  (c) it is positive definite: $u^t A u \geq 0$ for any $u \in \mathbb{R}^N$ and $u^t A u = 0$ if and only if $u = 0$.

(2) Let us consider the finite difference discritization of Poisson equation with Neumann boundary condition.
  (a) Write out the 9×9 matrix A for $h = 1/2$.
  (b) Prove that in general the matrix corresponding to Neumann boundary condition is only semi-positive definite.
  (c) Show that the kernel of A consists of constant vectors: $Au = 0$ if and only if $u = c$.

(3) Implement finite difference methods for solving the Poisson equation $u'' = f$ with pure Dirichlet or pure Neumann boundary in the unit interval $(0, 1)$.
  (a) Discretize the domain into uniform grid.
  (b) Discretize the Δ operator for interior points.
  (c) Modify the equation for boundary points.
  (d) Use director solver in Matlab to compute the solution: `u = A\f`.
  (e) Choose a smooth function and substitue into Poisson equation to get a right hand function. Then compare your computation with the true solution in the maximal norm, i.e., compute $\|u_I - u_h\|_\infty$.