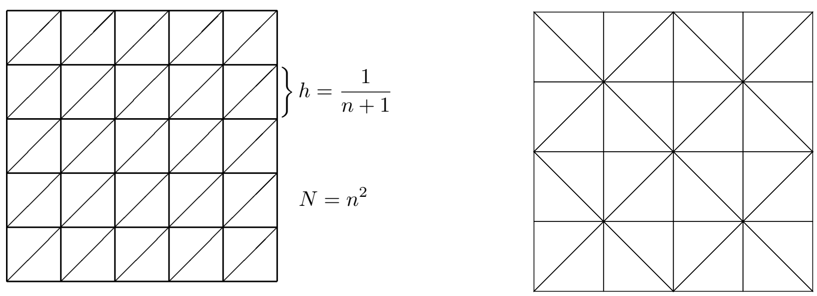
HOMEWORK 2 OF MATH 226A FALL 2009
DUE DATE: OCT 21

(1) Let $\boldsymbol{n}_i$ be the unit outward normal vector of the face F_i and d_i be the distance from $\boldsymbol{x}_i$ to F_i. Prove
$$\nabla \lambda_i = -\frac{1}{d_i}\boldsymbol{n}_i.$$

(2) Prove that, with central finite difference methods with five point stencil for Dirichlet boundary condition, the resulting stiffness matrix, when properly scaled, is the same as the linear finite element discretization on either three-directional uniform grids or criss-cross uniform grids.
Hint: Using formula in (1) to compute the local stiffness of the isosceles right triangle.



(a) xxxCaption 1xxx

(b) Three-directional Uniform Grids

FIGURE 1. Cirss-Cross Uniform Grids

(3) Let $e \in \mathcal{E}(\mathcal{T})$ be an interiori edge in the triangulation with nodes x_i and x_j, and shared by two triangles τ_1 and τ_2. Denoted the angle in τ opposing to e by θ_E^{τ}.
(a) Derive the following identity
$$a_{ij} = -\frac{1}{2}(\cot\theta_E^{\tau_1} + \cot\theta_E^{\tau_2}).$$
(b) Prove that $a_{ij} \leq 0$ if and only if the following Delaunay condition is satisfied:
$$\theta_E^{\tau_1} + \theta_E^{\tau_2} \leq \pi$$
(c) † Prove that if the Delaunay condition is satisfied for all interior edges in the triangulation (such a triangulation is called Delaunay triangulation), the finite element solution is nonnegative for the equation $-\Delta u = f$ (with homogeneous Dirichlet boundary condition) if $f \geq 0$.