Nonlinear Analysis
A workshop in Celebration of Herbert Amann's 85th Birthday
Program and Abstracts

Program

Thursday, May 23 2024		
12:30	Lunch Available	
15:00-15:30	H. Kozono	*Liouville-type theorems for the Taylor-Couette-Poiseuille flow of the stationary NSEs*
15:45:16:15	D. Daners	*The abstract logistic equation on rough domains*
16:15-17:00	Coffee Break	
17:00-17:30	E. Schrohe	*Asymptotics of the porous medium equation on manifolds with conical singularities*
17:45-18:15	P. Quittner	*Liouville theorems and universal estimates for superlinear problems*
18:30	Dinner	
Friday, May 24 2024		
9:00-9:30	L. Weis	*The Absolute Functional Calculus and Regularity Estimates for Evolution Equations*
9:45-10:15	M. Veraar	*Functional calculus of the Laplacian on weighted Sobolev spaces*
10:15-11:00	Coffee Break	
11:00-11:30	K. Pileckas	*Non-stationary Navier-Stokes equations in 2D power-cusp domain*
12:30	Lunch	
14:00-14:30	S. Shimizu	*Free boundary problems of the Navier–Stokes equations in the critical Besov space*
14:45-15:15	H. Abels	*Regularity and Convergence to Equilibrium for a NS-Cahn-Hilliard System with Unmatched Densities*
15:15-16:00	Coffee Break	
16:00-16:30	W. Arendt	*Maximal Regularity for Non-Autonomous Evolution Equations*
17:00-18:30	Reception	
18:30	Dinner	
Saturday, May 24 2024		
9:00-9:30	J. López-Gómez	*Universal a priori bounds for positive solutions of a class of superlinear indefinite problems*
9:45-10:15	Y. Shao	*On a thermodynamically consistent model for magnetoviscoelastic fluids in 3D*
10:15-11:00	Coffee Break	
11:00-11:30	J. Hernandez	*Beyond the classical strong maximum principle: sign-changing forcing term and flat solutions*
12:30	Lunch Available	

Abstracts

Helmut Abels

Regularity and Convergence to Equilibrium for a Navier-Stokes-Cahn-Hilliard System with Unmatched Densities

We study the initial-boundary value problem for an incompressible Navier-Stokes-Cahn-Hilliard system with non-constant density proposed by Abels, Garcke and Grün in 2012. This model arises in the diffuse interface theory for binary mixtures of viscous incompressible fluids. This system is a generalization of the well-known model H in the case of fluids with unmatched densities. In three dimensions, we prove that any global weak solution (for which uniqueness is not known) exhibits a propagation of regularity in time and stabilizes towards an equilibrium state as time tends to infinity. Our analysis hinges upon the following key points: a novel global regularity result (with explicit bounds) for the Cahn-Hilliard equation with divergence-free velocity belonging only to the Leray-Hopf class, the energy dissipation of the system, the separation property for large times, a weak strong uniqueness type result, and the Lojasiewicz-Simon inequality. This is a joint work with Harald Garcke and Andrea Giorgini. Finally, we comment on how these results are extended to the case of more than two fluids, which is a joint work with Harald Garcke and Andrea Poiatti.

Wolfgang Arendt

Maximal Regularity for Non-Autonomous Evolution Equations

The talk will be a survey on well-posedness results for non-autonomous evolution equations. We start by a Theorem of H. Amann in the situation where the domain of the generator is time independent and which holds on arbitrary Banach spaces. Then we will concentrate on the Hilbert space case. Here an elegant theorem by Lions is in the centre which gives maximal regularity however not in the desired space (where the domain is indeed constant). The problem on maximal regularity in the good space has a long history with several recent contributions. We will elaborate the relation to the Kato problem and also discuss results for elliptic operators. Part of the talk is based on a paper joint with D. Dier and S. Fackler.

Daniel Daners

The abstract logistic equation on rough domains

We consider the existence and uniqueness of solutions to the abstract logistic equation

$$-Au = \lambda u - m(x)g(u)u,$$

in $L^p(\Omega)$, where $-A$ is the generator of a compact irreducible positive analytic semi-group with some interior smoothing properties, $\Omega \subset \mathbb{R}$ with no or very mild regularity properties, $\lambda \geq 0$ a parameter, $g \in C^1([0,\infty))$ strictly increasing with $g(0) = 0$ and $m(x) \geq 0$ bounded and not identically zero. In the classical theory, the usual sub-and super-solution methods relies on Hopf's boundary maximum principle, essentially forcing C^2-regularity of Ω. The aim of this work is to replace that maximum principle by tools that do not rely on the boundary regularity, but only interior regularity. The tool is Kato's inequality that allows to prove a comparison theorem that is independent of any boundary conditions. This is joint work with Wolfgang Arendt.

Jesús Hernandez

Beyond the classical strong maximum principle: sign-changing forcing term and flat solutions

Almost a century ago, E. Hopf proved, in 1927, the *strong maximum principle* which, in a simple formulation, says that if Ω is a smooth bounded domain in $\mathbb{R}^N$ and

$$\begin{cases} -\Delta u \geq f(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

then

$$u(x) > 0 \text{ in } \Omega, \tag{P$_u$}$$

and

$$\frac{\partial u}{\partial n} < 0 \text{ on } \partial\Omega, \tag{2}$$

assumed that

$$f(x) \geq 0 \text{ in } \Omega. \tag{P$_f$}$$

In particular, under (P$_f$) we get the uniforme estimate

$$u(x) \geq C \left(\int_\Omega f(x) d(x, \partial\Omega) dx \right) d(x, \partial\Omega) \text{ a.e. } x \in \Omega, \tag{3}$$

for some $C > 0$ only dependent on Ω (Morel-Oswald (1987), Brezis-Cabré (1998)).

The main goal of this lecture is to show that the sign assumption (P$_f$) can be removed in a suitably way (for instance when $f(x) < 0$ in some neighborhood of $\partial\Omega$). Under such type of new assumptions on f we can prove:
(A) the positivity of u, property (P$_u$), still holds and $\frac{\partial u}{\partial n} \leq 0$ on $\partial\Omega$,
(B) under additional conditions on $f(x)$, the positive *solution* of the problem (1) [i.e., now with the equality symbol $=$, instead $\geq$] does not satisfy (2) but $\frac{\partial u}{\partial n} = 0$ on $\partial\Omega$ (this corresponds to the notion of *flat solution* already considered by different authors in the framework of some nonlinear problems (see, e.g., Díaz-Hernández-Ilyasov (2015)). This also shows that assumption (P$_f$) is necessary to conclude (3).

Applications to some *sublinear indefinite* semilinear equations and the linear heat equation $u_t - \Delta u = f(x,t)$ are also given.

The talk is based on joint work with J.I. Díaz.

Hideo Kozono

Liouville-type theorems for the Taylor-Couette-Poiseuille flow of the stationary Navier-Stokes equations

We study the stationary Navier–Stokes equations in the region between two rotating concentric cylinders. We first prove that, under the small Reynolds number, if the fluid is axisymmetric and if its velocity is sufficiently small in the L^∞-norm, then it is necessarily the Taylor-Couette-Poiseuille flow. If, in addition, the associated pressure is bounded or periodic in the z-axis, then it coincides with the well-known *canonical* Taylor-Couette flow. We discuss the relation between uniqueness and stability of such a flow in terms of the Taylor number in the case of narrow gap of two cylinders. The investigation in comparison with two Reynolds numbers based on inner and outer cylinders rotational velocities is also studied. Next, we give a certain bound of the Reynolds number and the L^∞-norm of the velocity such as the fluid is indeed, *necessarily axisymmetric*. As the result, it is clarified that smallness of Reynolds number of the fluid in the two rotating concentric cylinders governs both axisymmetry and the Taylor-Couette-Poiseuille flow with the exact form of the pressure.

This is based on the joint work with Profs. Yutaka Terasawa (Nagoya Univ.) and Yuta Wakasugi (Hiroshima Univ.).

Julián López-Gómez

Universal a priori bounds for the positive solutions of a class of superlinear indefinite problems

In this talk I plan to discuss some of my pioneering results in collaboration with H. Amann (JDE 1998), as well as to present some recent improvements in collaboration with J. L. Sampedro.

Konstantin Pileckas

Non-stationary Navier-Stokes equations in $2D$ power-cusp domain

The initial boundary value problem for the non-stationary Navier-Stokes equations is studied in $2D$ bounded domain with a power cusp singular point O on the boundary. The case of the boundary value with a nonzero flow rate is considered. In this case there is a source/sink in O and the solution necessary has infinite energy integral.

To find a solution, we first construct the formal asymptotic expansion $(\mathbf{U}^J, P^J)$ of it near the singular point, and then we find a solution in the form $\mathbf{u} = \zeta \mathbf{U}^J + \mathbf{v}$, where ζ is cutoff function and $\mathbf{v}$ has finite dissipation of energy.

Pavol Quittner

Liouville theorems and universal estimates for superlinear problems

It is known that Liouville-type theorems guarantee universal estimates of solutions to various superlinear elliptic and parabolic problems which are (at least asymptotically) scale-invariant. We discuss several recent related results, including results for problems without scale invariance.

This is a joint work with Philippe Souplet.

Yuanzhen Shao

On a thermodynamically consistent model for magnetoviscoelastic fluids in 3D

In this talk, we consider a system of equations that model a non-isothermal magnetoviscoelastic fluid, which is thermodynamically consistent. The system is analyzed by means of the L_p-maximal regularity theory. First, we will discuss the local existence and uniqueness of a strong solution. Then it will be shown that a solution initially close to a constant equilibrium exists globally and converges to a (possibly different) constant equilibrium. Finally, we will show that that every solution that is eventually bounded in the topology of the natural state space exists globally and converges to the set of equilibria. This is a joint work with Hengrong Du and Gieri Simonett.

Elmar Schrohe

Asymptotics of the porous medium equation on manifolds with conical singularities

The Porous Medium Equation (PME) is a non-linear variant of the heat equation. The problem is to find a solution u to the equations

$$\dot{u}(t,x) - \Delta u^m(t,x) = f(t,u), \quad u(0,x) = u_0(x).$$

The name is derived from the fact that it describes - among other phenomena - the flow of a gas in a porous medium. Here, u is the density of the gas, t is a time parameter, x the space variable, m is a positive constant, and f is a forcing term; for simplicity we assume it to be Lipschitz in t and holomorphic in u. Finally u_0 the value of u.

We consider the PME on a compact manifold with conical singularities for a strictly positive initial value u_0. It has been known for a long time that solutions to elliptic equations generically have singularities in a neighborhood of the conical points. More recently, it has become possible to determine the precise structure of the singularities that may occur.

In the talk I will explain, how maximal regularity techniques can be used to establish the existence of a solution in suitable cone Sobolev spaces with asymptotics, what we can say about the regularity of the solution and how the geometry of the manifold near the conical points is reflected in the structure of the solution.

(Joint work with N. Roidos, Patras/GR)

Senjo Shimizu

Free boundary problems of the Navier–Stokes equations in the critical Besov space

In this talk, we consider end-point maximal L^1-regularity upon the homogeneous Besov space for the Stokes equations with inhomogeneous stress boundary condition. We decompose the Fourier symbol of the Stokes equations in time and space regions. Utilizing the almost orthogonal properties between the boundary potential and the Littlewood–Paley decomposition, we show maximal L^1-regularity in the Besov and the Lizorkin–Triebel spaces. We further discuss an application to free boundary problems of the Navier–Stokes equations. This is a joint work with Takayoshi Ogawa (Waseda University).

Mark Veraar

Functional calculus of the Laplacian on weighted Sobolev spaces

Singularities of solutions to PDEs often appear at the boundary. It is well-known that one can use Sobolev spaces with weights such as $w(x) = \mathrm{dist}(x, \partial\Omega)^\gamma$ to handle such singularities. However, for $W^{k,p}$-regularity of solutions, one typically requires $\gamma = kp$. Unfortunately, this brings one outside the usual A_p-class of weights and new harmonic and functional analytic tools need to be developed to set up a regularity theory. In the talk, I will present several new results on the functional calculus of $-\Delta$ with Dirichlet and Neumann boundary conditions. Consequences for maximal regularity for elliptic and parabolic equations are also discussed, and in particular, several results of N. Krylov and K.-H. Kim are recovered and extended.

The talk is based on joint works with Nick Lindemulder, Emiel Lorist, and Floris Roodenburg.

Lutz Weis

The Absolute Functional Calculus and Regularity Estimates for Evolution Equations

Unavailable.