1. Let $f \in C(\mathbb{R}, \mathbb{R})$ satisfy
   
   \[ f(x + y) = f(x)f(y), \ x, y \in \mathbb{Q} \text{ and } f(0) = 1. \]

   What is $f$ and why?

2. Let $f \in C(\mathbb{R}, \mathbb{R})$ be periodic with period $T > 0$, that is,

   \[ f(x + T) = f(x) \quad \forall \ x \in \mathbb{R}. \]

   Show that $f$ is uniformly continuous.

3. Let $I \subset \mathbb{R}$ be an interval and assume that $f : I \to \mathbb{R}$ is monotone.

   Show that $f$ is continuous if $f(I)$ is also an interval.

4. Let $f \in C(K, \mathbb{R})$ for some compact set $K \subset \mathbb{R}$ and assume that $f > 0$. Show that $1/f$ is uniformly continuous.

5. Let $D_f, D_g \subset \mathbb{R}$, $f \in C(D_f, \mathbb{R})$, $g \in C(D_g, \mathbb{R})$ and assume that $g(D_g) \subset D_f$. Show that $f \circ g \in C(D_g, \mathbb{R})$.

Homework due by Thursday, November 10 2005.