Assignment 6

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1. Let $f \in \mathrm{C}(\mathbb{R},\mathbb{R})$ satisfy
$$f(x+y) = f(x)f(y)\,,\ x,y \in \mathbb{Q} \text{ and } f(0) = 1\,.$$
What is f and why?

2. Let $f \in \mathrm{C}(\mathbb{R},\mathbb{R})$ be periodic with period $T > 0$, that is,
$$f(x+T) = f(x)\,\forall\, x \in \mathbb{R}\,.$$
Show that f is uniformly continuous.

3. Let $I \subset \mathbb{R}$ be an interval and assume that $f : I \to \mathbb{R}$ is monotone. Show that f is continuous if $f(I)$ is also an interval.

4. Let $f \in \mathrm{C}(K,\mathbb{R})$ for some compact set $K \subset \mathbb{R}$ and assume that $f > 0$. Show that $1/f$ is uniformly continuous.

5. Let $D_f,\ D_g \subset \mathbb{R},\ f \in \mathrm{C}(D_f,\mathbb{R}),\ g \in \mathrm{C}(D_g,\mathbb{R})$ and assume that $g(D_g) \subset D_f$. Show that $f \circ g \in \mathrm{C}(D_g,\mathbb{R})$.

Homework due by Thursday, November 10 2005.