1. Let a function \( f \in \text{BUC}(\langle a, b \rangle, \mathbb{R}) \) be given for \( a < b \in \mathbb{R} \). Show that
\[
\lim_{x \to a^+} f(x), \lim_{x \to b^-} f(x)
\]
extist. Thus, \( f \) can be uniquely extended to a uniformly continuous function of the closed interval \([a, b]\).

2. Analyze the continuity and differentiability of the function
\[
f(x) := \begin{cases} 
0, & x \leq 0, \\
\frac{1}{x}e^{-\frac{1}{x^2}}, & x > 0.
\end{cases}
\]
What can you say about higher order derivatives at \( x = 0 \)?

3. Let \( f : D \to \mathbb{R} \) be differentiable at \( x_0 \in D \setminus \mathbb{R} \). Prove or disprove:
   (i) \( f \) is strictly increasing at \( x_0 \) implies that \( f'(x_0) > 0 \).
   (ii) \( f'(x_0) = 0 \) implies that \( f \) has a local minimum or maximum at \( x_0 \).
   (iii) \( f \) is Lipschitz continuous at \( x_0 \) and has local minimum there implies that \( f \) is differentiable at \( x_0 \).

4. Let \( a, b \in \mathbb{R} \) with \( a < b \) and \( f : [a, b] \to \mathbb{R} \). Define
\[
d^+ f(x_0) = \lim_{x \to x_0^+} \frac{f(x) - f(x_0)}{x - x_0}, \quad x_0 \in [a, b), \\
d^- f(x_0) = \lim_{x \to x_0^-} \frac{f(x) - f(x_0)}{x - x_0}, \quad x_0 \in (a, b],
\]
the right and left derivative of \( f \) at \( x_0 \), respectively, if the limit exists.
   (i) Show that \( f \) is differentiable at \( x_0 \in (a, b) \) if and only if \( d^+ f(x_0) \) and \( d^- f(x_0) \) exist and coincide.
   (ii) Assume that \( f \) takes on a minimum or a maximum at \( a \) or \( b \) and is one-side differentiable there. What can you say about \( d^+ f(a) \) and \( d^- f(b) \)?

5. Let \( f \in C^1(\mathbb{R}, \mathbb{R}), g \in C(\mathbb{R}, \mathbb{R}) \) and assume that \( f(x_0) = 0 \) for some \( x_0 \in \mathbb{R} \). Prove that \( fg : \mathbb{R} \to \mathbb{R}, x \mapsto f(x)g(x) \) is differentiable at \( x_0 \) and compute \( (fg)'(x_0) \). What can you say if \( g \) is only assumed to be bounded?