

## Assignment 11

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1. Let the function  $f : [0, 1] \rightarrow \mathbb{C}$  be given by

$$f(x) = \begin{cases} xe^{2i\pi/x}, & x \neq 0 \\ 0 & x = 0. \end{cases}$$

Show that  $f \in C([0, 1], \mathbb{C})$  and plot  $f([0, 1])$ .

2. Determine the convergence of the following series:

$$\sum_{k=1}^{\infty} \frac{e^{i\frac{\pi}{2}k}}{k}, \quad \sum_{k=1}^{\infty} (-1)^k (\sqrt{k+1} - \sqrt{k}), \quad \sum_{k=1}^{\infty} \frac{k!}{k^k}.$$

3. Let  $(x_n)_{n \in \mathbb{N}}$  be decreasing and assume that  $\sum_{k=1}^{\infty} x_k$  converges. Prove that  $\lim_{k \rightarrow \infty} kx_k = 0$ .
4. Prove the root and ratio tests from class.
5. Give an example of  $(a_{mn})_{m,n \in \mathbb{N}}$  for which

$$\sum_{m=1}^{\infty} \left( \sum_{n=1}^{\infty} a_{mn} \right) \neq \sum_{n=1}^{\infty} \left( \sum_{m=1}^{\infty} a_{mn} \right).$$