Assignment 14

1. Let the function $f \in C(\mathbb{R}, \mathbb{R})$ be given by
$$f(x) = \begin{cases} 1 - |x|, & |x| < 1, \\ 0, & |x| \geq 1. \end{cases}$$
Compute $f * f$ and $f * f * f$.

2. Show that $f : \mathbb{R} \to \mathbb{R}$, $x \mapsto \frac{1}{1+x^2}$ is analytic.

3. Let $f \in C_c(\mathbb{R}, \mathbb{R})$ and $g \in C^1(\mathbb{R}, \mathbb{R})$. Show that
$$f * g \in C^1(\mathbb{R}, \mathbb{R}).$$

4. For $f \in C_c(\mathbb{R}, \mathbb{R})$ define its Fourier transform $\hat{f} : \mathbb{R} \to \mathbb{K}$ by
$$\hat{f}(\xi) = \int_{-\infty}^{\infty} e^{-ix\xi} f(x) \, dx.$$
Show that $\hat{f}$ is well-defined and analytic. Give an estimate for the radius of convergence of its power series expansion about $\xi = 0$.

5. You ask a question.

The Homework is due on Friday, February 28.