# Assignment 17

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1. Prove continuity of the mapping
$$K : \mathrm{C}\big([0,1],\mathbb{R}^n\big) \to \mathrm{C}\big([0,1],\mathbb{R}^n\big)\,,\ f \mapsto K(f)$$
defined through
$$K(f)(x) := \int_0^x f(y)\,dy\,,\ x \in [0,1]\,.$$

2. Let $(M, d_M)$ be a metric space. Show that
$$d_{x_0} : M \to \mathbb{R}\,,\ x \mapsto d_M(x, x_0)$$
is continuous.

3. Let $(M, d_M)$ and $(N, d_N)$ be metric spaces and assume that $N$ is complete. Define $d_{\mathrm{B}(M,N)}$ through
$$d_{\mathrm{B}(M,N)}(f,g) = \sup_{x\in M} d_N\big(f(x), g(x)\big)\,,\ f,g \in \mathrm{B}(M,N)$$
and prove that
$$\big(\mathrm{B}(M,N), d_{\mathrm{B}(M,N)}\big)$$
is complete.

4. Formulate and prove the fact that the composition $f \circ g$ of continuous functions $f, g$ defined on metric spaces is continuous and give an example where the composition is continuous but nor $f$ or $g$ is continuous.

5. You ask a question.

The Homework is due Friday, April 11.