1. Find an example of a connected set which is not pathwise connected.

2. Assume that $x_0 \in \mathbb{R}^n$ and that $f \in \mathcal{C}^1(\mathbb{R}^n, \mathbb{R}^n)$ is such that
   \[ |f(x) - f(y)| \leq L|x - y|, \quad x, y \in \mathbb{R}^n \]
   for some $L < 1$. Show that the operator
   \[ T : \mathcal{C}([0, 1], \mathbb{R}^n) \to \mathcal{C}([0, 1], \mathbb{R}^n), \quad u \to Tu \]
   defined through
   \[ (Tu)(t) = x_0 + \int_0^t f(u(\tau)) \, d\tau, \quad t \in [0, 1] \]
   has a fixed point in $\mathcal{C}([0, 1], \mathbb{R}^n)$ which solves
   \[ \begin{cases} 
   u' = f(u) \\
   u(0) = x_0 
   \end{cases} \]

3. Let $f \in \mathcal{C}(D, \mathbb{R}^m)$ for some $D \subset \mathbb{R}^n$ and assume it is differentiable at $x \in D$. Show that there exists a constant $M > 0$ such that
   \[ |f(x) - f(y)| \leq M|x - y| \]
   for $y$ in a neighborhood of $x$.

4. Let $f \in \mathcal{C}^1(D, \mathbb{R})$ for some $D \subset \mathbb{R}^n$. Fix $x \in D$ and assume that $\nabla f(x) \neq 0$. Show that $\nabla f(x)$ points in the direction of maximal growth of $f$.

5. You ask a question.

The Homework is due Friday, April 18.