1. Consider the function \( f : \mathbb{R}^2 \to \mathbb{R} \) given by
\[
f(x, y) = \begin{cases} 
\frac{xy(x^2 - y^2)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\
0, & (x, y) = (0, 0)
\end{cases}
\]
and compute its mixed second derivatives.

2. Denote \( \mathbb{R}^n \setminus \{0\} \) by \( \mathbb{R}^n \). A function \( f : \mathbb{R}^n \to \mathbb{R} \) is called homogeneous of degree \( k \) if
\[
f(tx) = t^k f(x), \quad t > 0, \quad x \in \mathbb{R}^n.
\]
Show that
\[
\nabla f(x) \cdot x = kf(x)
\]
if \( f \) is differentiable.

3. Let \( f \in C^1(D, \mathbb{R}) \) for some convex \( D \subseteq \mathbb{R}^n \) and \( x, y \in D \). Show that there exists \( \xi \in D \) such that
\[
f(y) - f(x) = \nabla f(\xi) \cdot (y - x).
\]

4. Show that the existence of all partial derivatives for \( f : \mathbb{R}^n \to \mathbb{R} \) at a point \( x \in \mathbb{R}^n \) does not imply its differentiability there.

5. You ask a question.