Assignment 19

1. Consider the function $f:\mathbb{R}^2\to\mathbb{R}$ given by
$$f(x,y)=\begin{cases}\frac{xy(x^2-y^2)}{x^2+y^2}\,, & (x,y)\neq(0,0)\\ 0\,, & (x,y)=(0,0)\end{cases}$$
and compute its mixed second derivatives.

2. Denote $\mathbb{R}^n\backslash\{0\}$ by $\dot{\mathbb{R}}^n$. A function $f:\dot{\mathbb{R}}^n\to\mathbb{R}$ is called homogeneous of degree k if
$$f(tx)=t^kf(x)\,,\ t>0\,,\ x\in\dot{\mathbb{R}}^n\,.$$
Show that
$$\nabla f(x)\cdot x=kf(x)$$
if f is differentiable.

3. Let $f\in \mathrm{C}^1(D,\mathbb{R})$ for some convex $D\overset{o}{\subset}\mathbb{R}^n$ and $x,y\in D$. Show that there exists $\xi\in D$ such that
$$f(y)-f(x)=\nabla f(\xi)\cdot(y-x)\,.$$

4. Show that the existence of all partial derivatives for $f:\mathbb{R}^n\to\mathbb{R}$ at a point $x\in\mathbb{R}^n$ does not imply its differentiability there.

5. You ask a question.

The Homework is due Friday, April 25.