

## Assignment 20

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1. Classify the critical points of

$$f : \mathbb{R}^2 \rightarrow \mathbb{R}, (x, y) \mapsto x^2 + y^2 - x^4 - y^4.$$

2. Let  $f \in C^3(\mathbb{R}^n, \mathbb{R})$  and  $x \in \mathbb{R}^n$  be such that

$$f(x) = 0, \nabla f(x) = 0, D^2f(x) = 0.$$

Assuming that there is  $\alpha \in \mathbb{N}^n$  with  $|\alpha| = 3$  and  $\partial^\alpha f(x) \neq 0$ , what can you say about the critical point  $x$ ?

3. Let  $B_f$  be the body of revolution obtained by rotating the graph of  $f \in C([-1, 1], (0, \infty))$  about the  $x$ -axis in  $\mathbb{R}^3$ . Compute its volume. [Hint: Change of variables.]

4. Compute  $\nabla f$  for

$$f : \mathbb{R}^{n \times n} \rightarrow \mathbb{R}, A \mapsto \det(A)$$

at  $A = E_n$  where  $E_n$  is the identity matrix.

5. You ask a question.

The Homework is due Friday, May 1.