Assignment 21

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1. Let $f, g \in C^1(\mathbb{R}^2, \mathbb{R})$ be given. Find necessary conditions for the existence of $u \in C^2(\mathbb{R}^2, \mathbb{R})$ such that
$$\nabla u = (f, g)\,.$$
Show that those conditions are also sufficient and derive a formula describing u in terms of f and g.

2. Let $F \in C^2(D, \mathbb{R}^n)$ for some $D \overset{o}{\subset} \mathbb{R}^n$. In order to solve $F(x) = 0$ one can use Newton iteration
$$x_{n+1} = x_n - DF(x_n)^{-1} F(x_n)\,,$$
with some initial guess $x_0 \in D$. Let $z \in D$ be a zero of F and prove that $(x_n)_{n \in \mathbb{N}}$ converges to z if the initial guess is chosen close enough to z.

3. Consider the system
$$\begin{cases} x^2 + y^2 - u^2 - xv &= 0 \\ xy + uv - v^2 &= 0 \end{cases}$$
and determine the points $(x, y, u, v) \in \mathbb{R}^4$ where the system can be solved for (u, v) for sure.

4. The level sets (curves) $L_c(f)$ of a function $f \in C^1(\mathbb{R}^2, \mathbb{R})$ are defined by
$$L_c(f) := \{x \in \mathbb{R}^2 \mid f(x) = c\}\,,\ c \in \mathbb{R}\,.$$
Show that $\nabla f(x)$ is perpendicular to $L_c(f)$ at the point x for all $x \in L_c(f)$ such that $\nabla f(x) \neq 0$.

5. You ask a question.

The Homework is due Friday, May 9.