1. Let \((M, d_M)\) and \((N, d_N)\) be metric spaces. Show that
\[ f \in C(M, N) \iff f(A) \subset f(A), \ A \subset M. \]

2. Let \(f : (-1, 1) \to \mathbb{R}\) be such that
\[ f(0) = 0 \text{ and } f(x) \geq c|x|^\alpha, \ x \in (-1, 1) \]
for some \(\alpha \in (0, 1)\) and some \(c > 0\). Conclude that \(f\) is not differentiable at \(x = 0\).

3. Show that \(O(n) := \{M \in \mathbb{R}^{n \times n} | MM^T = 1_n\}\) is a compact subset of \(\mathbb{R}^{n \times n}\).

4. Show that the series
\[ \sum_{n=1}^{\infty} (1 - \cos(x/n)) \]
converges uniformly on compact subsets of \(\mathbb{R}\).

The Homework is due Friday, May 30.