1. Let $A = (a_{jk})_{j,k \in \mathbb{N}}$ be a double array of real numbers and let 
\[ d = (d_1, d_2, d_3, \cdots) = (a_{11}, a_{21}, a_{12}, a_{31}, a_{22}, a_{13}, \cdots) \]
be the sequence obtained by concatenating the finite diagonals
\[ d_m = (a_{m1}, a_{m-12}, \cdots, a_{1m}), \quad m \in \mathbb{N}. \]
Show that any limit point of any row $A_{j\cdot} = (a_{jk})_{k \in \mathbb{N}}$ or column $A_{\cdot k} = (a_{jk})_{j \in \mathbb{N}}$ of $A$ is also a limit point of the sequence $d$. Do we obtain all limit points of $d$ this way?

2. Let $A \subset \mathbb{R}$. We say that $B \subset A$ is open in $A$, or, concisely $B \circ \subset A$, iff there is an open set $\tilde{B} \subset \mathbb{R}$ with $B = A \cap \tilde{B}$.

Show that
\begin{enumerate}
  \item $(\emptyset, A \circ) \subset A$.
  \item If $B_j \circ \subset A$ for $j \in \mathbb{N}$, then $\bigcup_{j \in \mathbb{N}} B_j \circ \subset A$.
  \item If $B_j \circ \subset A$ for $j = 1, \ldots, m$ ($m \in \mathbb{N}$), then $\bigcap_{1 \leq j \leq m} B_j \circ \subset A$.
\end{enumerate}

Is $[0, 1/2)$ open in $[0, 1]$? What about $(0, 1/2]$? Is $\{0\}$ open in $\mathbb{N}$? Is it open in $\mathbb{Q}$?

3. Let $x \in \mathbb{R}^\mathbb{N}$ and let
\[ X = \{y \in \mathbb{R} \mid y = x_j \text{ for some } j \in \mathbb{N}\}. \]
What is the relation between the limit points of the sequence $x$ and those of the set $X$?

4. Let $A \subset \mathbb{R}$. The sets $\overline{A}$, $\overset{o}{A}$ and $LP(A)$ were defined in class. Let, in addition, $\partial A = \overline{A} \setminus A$. Prove or disprove the following:
\begin{align*}
\overset{o}{A} & \subset \overline{A}, \quad \overline{A} = LP(A), \\
LP(A) & \subset A, \quad LP(LP(A)) \subset LP(A), \\
LP(A) & \subset LP(LP(A)), \quad \overline{\partial A} = \partial A, \\
\overline{A} & = LP(A) \cup A, \quad \partial(\partial A) = \partial A.
\end{align*}

The Homework is due Friday, November 1