1. Let \( A \subset \mathbb{R} \) be a closed set. Construct a function \( f \in C^1(\mathbb{R}, \mathbb{R}) \) such that
\[
f(x) = 0 \iff x \in A.
\]
2. Let \( k \in \mathbb{N} \) and show that
\[
f : (0, \infty) \to \mathbb{R}, \ x \mapsto x^{1/k}
\]
can be defined as the inverse of \( g : (0, \infty) \to \mathbb{R}, \ x \mapsto x^k \) and compute \( f' \).
3. Show that there exists a real real-valued continuously differentiable function which is invertible and for which the inverse is not Hölder continuous of any exponent.
4. Let \( f \in C^2((a, b), \mathbb{R}) \) and assume that \( f''(x) \geq 0, \ x \in (a, b) \). Prove that
\[
f(tx + (1 - t)y) \leq tf(x) + (1 - t)f(y), \ t \in [0, 1], \ x, y \in (a, b).
\]