Math205a FALL TERM 2002

Assignment 8

---

1. Let $A \subset \mathbb{R}$ be a closed set. Construct a function $f \in \mathrm{C}^1(\mathbb{R}, \mathbb{R})$ such that
$$f(x) = 0 \iff x \in A\,.$$
2. Let $k \in \mathbb{N}$ and show that
$$f : (0, \infty) \to \mathbb{R}\,,\ x \mapsto x^{1/k}$$
can be defined as the inverse of $g : (0, \infty) \to \mathbb{R}\,, x \mapsto x^k$ and compute f'.

3. Show that there exists a real real-valued continuously differentiable function which is invertible and for which the inverse is not Hölder continuous of any exponent.

4. Let $f \in \mathrm{C}^2\big((a, b), \mathbb{R}\big)$ and assume that $f''(x) \geq 0\,,\ x \in (a, b)$. Prove that
$$f\big(tx + (1 - t)y\big) \leq tf(x) + (1 - t)f(y)\,,\ t \in [0, 1]\,,\ x, y \in (a, b)\,.$$

The Homework is due Wednesday December 4 2002