1. [Least squares] Given points \((x_i, y_i) \in \mathbb{R}^2, i = 1, \ldots, n\), find the straight line \(y = mx + b\) that best fits these points in that it minimizes

\[
f(m, b) := \sum_{i=1}^{n} (mx_i + b - y_i)^2.
\]

Show that such a line always exists regardless of the given set of points.

2. Find the points on the ellipse \(x^2 + 2y^2 = 1\) closest to and farthest from the origin.

3. Find an equation for the line tangent to the intersection curve of the surfaces \(x^3 + 3x^2y^2 + y^2 + 4xy - z^2 = 0\) and \(x^2 + y^2 + z^2 = 11\) at the point \((1, 1, 3)\).

4. Consider the function

\[
f : \mathbb{R}^2 \rightarrow \mathbb{R}, \quad (x, y) \mapsto x^3 + xy + y^3 + 1
\]

and find the points \((x, y) \in \mathbb{R}^2\) for which the set \(f^{-1}(f(x, y))\) is a 1-dimensional manifold embedded in \(\mathbb{R}^2\). Draw the set of points for which that is not the case.

5. Give a quadratic approximation to \(f(x, y) = e^x \cos(y), (x, y) \in \mathbb{R}^2\), at the origin.

Not due.