Assignment 18

1. Compute the gradient of the functions defined by
$$f(x,y)=\begin{cases} x^2y^2\log(x^2+y^2)\,, & (x,y)\neq(0,0)\\ 0\,, & (x,y)=(0,0)\end{cases}$$
$$g(x,y)=\begin{cases} xy\sin(\frac{1}{x^2+y^2})\,, & (x,y)\neq(0,0)\\ 0\,, & (x,y)=(0,0)\end{cases}$$
where it exists.

2. Assume that
$$f(y)=b+A(y-x)+o(|x-y|_2)\text{ as }y\to x$$
for some $f:\mathbb{R}^n\to\mathbb{R}^m$ with $m,n\geq 1$, $b\in\mathbb{R}^m$ and $A\in\mathbb{R}^{m\times n}$. Show that b and A are uniquely determined by the validity of the asymptotic relation.

3. Let $f\in \mathrm{C}^1(D,\mathbb{R}^m)$ for some $D\overset{o}{\subset}\mathbb{R}^n$. Show that there exists a constant $M>0$ such that
$$|f(x)-f(y)|\leq M\,|x-y|$$
for y in a neighborhood of x.

4. Let $f\in \mathrm{C}^1(D,\mathbb{R})$ for some $D\overset{o}{\subset}\mathbb{R}^n$. Fix $x\in D$ and assume that $\nabla f(x)\neq 0$. Show that $\nabla f(x)$ points in the direction of maximal growth of f.

5. Consider the map
$$\phi:\mathbb{R}^{n\times n}\to\mathbb{R}^{n\times n}\,,\;A\mapsto A^3$$
and show that it is differentiable. What is its derivative $D\phi(A)$ at a point A? Assume $n>1$.

Homework due by Thursday, April 20 2006