Assignment 23

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1. Let a unit circle C roll on the x-axis from left to right. Find a parametrization for the trajectory followed by any arbitrary point $P \in C$ during this motion.

2. Find a parametric representation for a "circular doughnut" (torus).

3. Give concrete examples for all possible ways an immersion can fail to be an embedding.

4. Let $M_m \subset \mathbb{R}^n$ be an m-dimensional C^1-manifold. Show that the tangent space to M_m at any point $x \in M_m$ does not depend on the choice of local representation (parametrization) g for the manifold.

5. Prove that the *unit sphere*
$$\mathbb{S}^{n-1} = \left\{ x \in \mathbb{R}^n \,\middle|\, |x|_2 = 1 \right\}$$
is a $(n-1)$-dimensional C^1-manifold in $\mathbb{R}^n$. Characterize its tangent space at every point.

Homework due by Thursday, May 25 2006