Midterm Examination

Print your name: ___________________  ___________________

Print your ID #: ________________________________

You have 50 minutes to solve the problems. Good luck!
1. Show that $\mathbb{R} \setminus \{0\}$ and $\mathbb{R}$ have the same cardinality by explicitly constructing a bijection between them.
2. Given a sequence \((x_n)_{n \in \mathbb{N}}\) of reals, define

\[ A := \{ x \in \mathbb{R} \mid x \text{ is an accumulation point of } (x_n)_{n \in \mathbb{N}} \}. \]

Prove that \(A\) is closed.
3. Let two subsets $A, B \subseteq \mathbb{R}$ be given. Show that

$$A \cap B \subseteq \overline{A} \cap \overline{B}$$

and give an example which shows that the inclusion is proper in general.
4. Let $C_1$ and $C_2$ be compact subsets of $\mathbb{R}$. Prove that

$$C_1 + C_2 := \{x + y \mid x \in C_1, \ y \in C_2\}$$

is compact, too.
5. Let $f : \mathbb{R} \to \mathbb{R}$ be uniformly continuous. Is $f^2$ continuous? Is it uniformly continuous? If your answer is yes, give a proof. If it is no, give a counterexample.