MATH 205 FALL TERM 2005

Midterm Examination

Print your name: _________________ _________________

Print your ID #: ____________________________________

You have 50 minutes to solve the problems. Good luck!

1. Show that $\mathbb{R} \setminus \{0\}$ and $\mathbb{R}$ have the same cardinality by explicitly constructing a bijection between them.

2. Given a sequence $(x_n)_{n\in\mathbb{N}}$ of reals, define

$$A := \big\{ x \in \mathbb{R} \,\big|\, x \text{ is an accumulation point of } (x_n)_{n\in\mathbb{N}} \big\}.$$

Prove that A is closed.

3. Let two subsets $A, B \subset \mathbb{R}$ be given. Show that

$$\overline{A \cap B} \subset \overline{A} \cap \overline{B}$$

and give an example which shows that the inclusion is proper in general.

4. Let C_1 and C_2 be compact subsets of $\mathbb{R}$. Prove that

$$C_1 + C_2 := \{x + y \,|\, x \in C_1\,,\ y \in C_2\}$$

is compact, too.

5. Let $f : \mathbb{R} \to \mathbb{R}$ be uniformly continuous. Is f^2 continuous? Is it uniformly continuous? If your answer is yes, give a proof. If it is no, give a counterexample.