1. Let $\mathbb{H}^n$ be the upper half-space. Given $m \in \mathbb{N}$ construct an extension operator $\text{ext} : C^m(\mathbb{H}^n) \to C^m(\mathbb{R}^n)$ such that 
\[ (\text{ext} u)|_{\mathbb{H}^n} = u, \; u \in C^m(\mathbb{H}^n). \]
Using localization arguments this result can be extended to cover extension from a domain with smooth boundary.

2. Let Banach spaces $E_j$, $j = 0, 1, 2$, be given with 
\[ E_2 \hookrightarrow E_1 \hookrightarrow E_0. \]
Show that, given $\varepsilon > 0$, there is a constant $c_\varepsilon > 0$ such that 
\[ \|u\|_{E_1} \leq \varepsilon \|u\|_{E_2} + c_\varepsilon \|u\|_{E_0}, \; u \in E_2. \]

3. Show that the trace operator $\gamma_{\partial \mathbb{H}^n}$ satisfies 
\[ \gamma_{\partial \mathbb{H}^n}(H^2(\mathbb{R}^n)) = H^{3/2}(\partial \mathbb{H}^n). \]

4. Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with smooth boundary, $\alpha > 0$ and $f \in L^2(\Omega)$. Find the boundary value problem for which
\[ \int_{\Omega} \nabla u \cdot \nabla v \, dx + \int_{\Omega} uv \, dx + \alpha \int_{\partial \Omega} uv \, d\sigma_{\partial \Omega} = \int_{\Omega} f v \, dx \; \forall v \in H^1(\Omega) \]
is the appropriate weak formulation and show that it possesses a unique solution.

5. Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with smooth boundary and find the natural weak $L^2(\Omega)$-formulation of the bvp
\[
\begin{aligned}
\Delta^2 u &= f \quad \text{in } \Omega, \\
u &= 0 \quad \text{on } \partial \Omega, \\
\partial_n v &= 0 \quad \text{on } \partial \Omega,
\end{aligned}
\]
and prove that it has a unique solution.