5-27-10

Homework

8.3

#16 A 4x6 non zero matrix

a) what is maximum rank

b) if rank $(A|\vec{b}) = 2$ for what ranks of A is $A\vec{x} = \vec{b}$ in consistant.

for what ranks of A is $A\vec{x} = \vec{b}$ inconsistant?

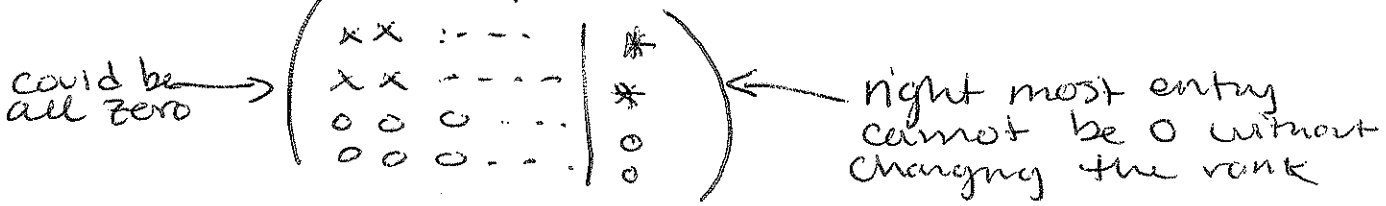
a) $\mathbb{R}^6 \longrightarrow \mathbb{R}^4$
$V \longmapsto AV$

maximum is 4, because the image is a subset of $\mathbb{R}^4$, so the rank is at most 4.

— side note.
if A was 4x3, the maximum rank would have been 3. $\mathbb{R}^3 \to \mathbb{R}^4$

b) $rk\left(A \middle| \begin{matrix} b_1 \\ \vdots \\ b_n \end{matrix}\right) = 2$

if I did row opps I would get



rank of A is 1, 2, or 0.
if rank of A is < 2, then we could get all zero's on another row except for the rightmost entry

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#17

$$A = \begin{pmatrix} 2 & 1 & 7 \\ 1 & 0 & 2 \\ -1 & 5 & 13 \end{pmatrix}$$

look at v_1, v_2, v_3 columns observe $2v_1 + 3v_2 - v_3 = 0$

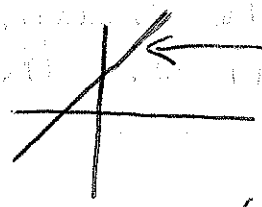
What can we say?

Answer: This is a linear combination of the columns which is zero, so v_1, v_2, v_3 are linearly dependent! rank < 3.

7.6

#2. look at the set $\{(a_1, a_2) \mid a_2 = 3a_1 + 1\} = L$

Is this a vector space?

NOT 

for this to be a vector space we need an addition and multiplication on it.

a vector space take $(0,1) + (0,1) = (0,2)$
$\notin L$

#11 – 16

look at $C(-\infty, \infty) = \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is continuous}\}$

Vector Space
– Add and multiplication by Scalar.

11. look at $W = \{f : \mathbb{R} \to \mathbb{R} \mid f \text{ is continuous} \mid f(1) = 0\}$
for W to be a subspace it needs to be closed under (+) and multiplication. i.e.
if $f \in W$ and $f_2 \in W$ then $f_1 + f_2 \in W$
and $\lambda f_1 \in W$ $\not\in$ (1,0) W. (yes) b/c $f_1 + f_2 @ 1 = f_1 @ 1 + f_2 @ 1 = 0 + 0$

#12. $W_2 = \{ f \in C \mid f(0) = 1 \}$

(no), not a subspace

$f(x) = x+1$ $\quad f \in W_2$

$0f = 0$ $\quad f \notin W_2$ b/c $= 0f(0) \neq 1$

Side note

$g(x) = 2x+1$

$f(x) = x$

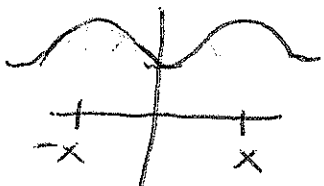
$(f+g)(x) = 3x+1$

$f(x) = 0$

$(f+g)(x) = g(x)$

So $f(x) = 0$ is the "origin" of C.

#14 $W_3 = \{ f \in C \mid f(-x) = f(x) \}$



Symmetric along y axis. "even function"

(yes) a subspace.

#15 $\{$differentiable functions$\}$

$(f_1 + f_2)' = f_1' + f_2'$. (yes)

#16 All fs of the form $C_1 e^x + C_2 x e^x$

take the span of $\{e^x, xe^x\}$

Span is always a subspace. (yes).

#20. Is $W = \{ f \in C \mid \int_a^b f \, dx = 0 \}$ a subspace.

$C[a,b]$ if f_1 and f_2 are in W

1) $f(1) = \int f_1 = 0$ $\quad \int (f_1 + f_2) = \int f_1 + \int f_2$

8.2

#20

$$\left(\begin{array}{cccc|c} 1 & 2 & 0 & 1 & 0 \\ 4 & 9 & 1 & 12 & 0 \\ 3 & 9 & 6 & 21 & 0 \\ 1 & 3 & 1 & 9 & 0 \end{array}\right)$$ ← can ignore (all 0's)

$$\begin{array}{l} R_4 - R_1 \\ R_2 - 4R_1 \\ R_3 - 3R_1 \end{array}\left(\begin{array}{cccc} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 8 \\ 0 & 3 & 6 & 18 \\ 0 & 1 & 1 & 8 \end{array}\right) \rightarrow \begin{array}{l} R_4 - R_2 \\ \frac{R_3}{3} \end{array}\left(\begin{array}{cccc} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 8 \\ 0 & 1 & 2 & 6 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\rightarrow R_3 - R_2 \left(\begin{array}{cccc} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 8 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

Upper triangular.
rank = 3
$\det A = 0$
infinite # solutions
nullity = 1 ⇒ 1 parameter
Soln Space to $A\vec{v} = 0$.

no inverse
$\det(A^{-1}) = \frac{1}{\det A}$ ← $\neq 0$