Name :

Math 240 Quiz 3 4

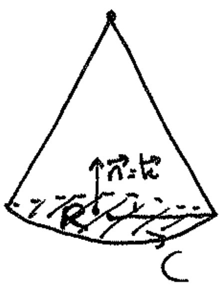
1. Let $\mathbf{F}$ be the vector field

$$\mathbf{F} = (2x - y, 2xz^2, \sin x)$$

Let S be the portion of the cone $z = 1 - \sqrt{x^2 + y^2}$ above the $z = 0$ plane; with outward pointing normal field $\mathbf{n}$.

Find the value of the integral

$$\int\int_S \mathrm{Curl}(F) \cdot \mathbf{n} dS$$



Use Stokes' theorem

$$\iint_S \mathrm{Curl}\,F \cdot n\, dS = \int_C F \cdot dr$$

Use Stokes' theorem again

$$\int_C F \cdot dr = \iint_R \mathrm{Curl}\,F \cdot \vec{n}$$

$\vec{n} = k$ so we only need the k component of Curl F

$$= \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA = \iint_R (2z^2 + 1)\, dA = \iint_R dA$$

$z = 0$ on R $\qquad = \pi$

2. Let C be a closed curve in $\mathbb{R}^3$. What should be the relationship between a and b so that we know (for sure) that

$$\int_C y^2 e^{bx} dx + aye^{bx} dy + sin(z) dz = 0$$

.

We must have $\text{Curl} \vec{F} = 0$

equivalently $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$ $\quad \underbrace{\frac{\partial Q}{\partial z} = \frac{\partial R}{\partial y} \qquad \frac{\partial P}{\partial z} = \frac{\partial R}{\partial x}}_{\text{true because these are all } 0}$.

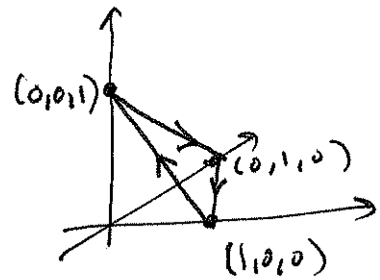
$\frac{\partial Q}{\partial x} = abye^{bx}$

$\frac{\partial P}{\partial y} = 2ye^{bx}$ so we need:

$abye^{bx} = 2ye^{bx}$

So $ab = 2$ $\qquad b = \frac{2}{a}$

3. Let $\mathbf{F} = (2x, 3z, y)$. Let C be the triangle whose vertices are $(1,0,0)$, $(0,0,1)$ and $(0,1,0)$ traversed in <u>that order</u>. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$



By Stokes' theorem,

$$\int_C F \cdot dr = \iint_S \operatorname{Curl} F \cdot \vec{n}$$

but $\vec{n}$ has to be pointing downward because of the orientation of C.

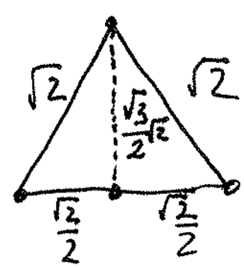
$$\vec{n} = \frac{1}{\sqrt{3}}(-1,-1,-1)$$

$$\operatorname{Curl} F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = (1-3)i + 0j + 0k$$

$$= -2i$$

$$\iint_{\text{triangle}} \operatorname{Curl} F \cdot \vec{n} = \iint_{\text{triangle}} \frac{2}{\sqrt{3}}\, dA = \frac{2}{\sqrt{3}} (\text{area of triangle})$$

$$= 2$$



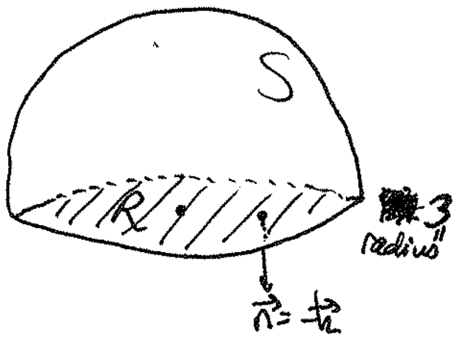
$\rightsquigarrow$ area $= \sqrt{3}$

$z+1$

4. Let S be the surface given by $x^2 + y^2 + z^2 = 9$ and $z \geq 0$. Let $\mathbf{F} = (2x, 3y,$ [redacted] and $\mathbf{n}$ the outward pointing normal vector field. Find $\int\int_S \mathbf{F} \cdot \mathbf{n}$

We use the divergence theorem. But the surface needs to be closed.

We have $\iint_S F \cdot \vec{n} + \iint_R F \cdot (-\vec{k}) = \iiint \operatorname{div} F$



$\operatorname{div} F = 2 + 3 + 1 = 6$

$$\iiint \operatorname{div} F = \left(\frac{\frac{4}{3}\pi(3)^3}{2}\right) \cdot 6 = 108\pi$$

$$\iint_R F \cdot (-\vec{k}) = \iint_R -(z+1) = -9\pi .$$

$z = 0$ on R

final answer: 108pi + 9pi = 117pi