MATH 3A: HOMEWORK 1

Due Tuesday, October 4, at the beginning of your discussion session

1. In Problems 1 and 2, for a given augmented matrix of a linear system, complete the row reduction procedure and describe the solution set:

$$\begin{pmatrix} 1 & -4 & -3 & 0 & 7 \\ 0 & 1 & 4 & 0 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 11 & -5 \end{pmatrix}.$$

2.

$$\begin{pmatrix} 1 & 3 & 0 & -2 & -7 \\ 0 & 1 & 0 & 3 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{pmatrix}.$$

3. Determine if the following system is consistent. You don't need to solve it.

$$2x_1 - 4x_4 = -10$$
$$3x_2 + 3x_3 = 0$$
$$x_3 + 4x_4 = -1$$
$$-3x_1 + 2x_2 + 3x_3 + x_4 = 5.$$

4. Find all values of h such that the following system is consistent

$$x_1 + hx_2 = -5$$
$$2x_1 - 8x_2 = 6.$$

5. Describe the equation on g, h, k that makes the following augmented matrix correspond to a *consistent* system.

$$\begin{pmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{pmatrix}.$$

6. Determine if the following matrices are in echelon or in reduced echelon form. Explain your answer.

a. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d. $\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$.

7. Find a general solution of the system with augmented matrix

$$\begin{pmatrix} 1 & 0 & -5 & 0 & -8 & 3 \\ 0 & 1 & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

8. Suppose that 3×5 *coefficient* matrix for a system has three pivot columns. Is the system consistent? Explain your answer.

9. Give an example of an *inconsistent* linear system in five variables with two equations.

10. True or false? Explain.
    (a) Any two matrices with the same number of rows are row equivalent.
    (b) Elementary operations on augmented matrices can change the solution set of the linear system.
    (c) A consistent system of linear equations has one or more solutions.