MATH 3A: HOMEWORK 6

Due Tuesday, Nov 22st, at the beginning of your discussion session

1. As R runs through each of the following six matrices: explain which row operation is performed when a matrix A is multiplied on the left by the matrix R; and compute $\det R$ and $\det RA$.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & k & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ k & 0 & 1 \end{bmatrix} \begin{bmatrix} k & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Compute the determinant of the following matrix. Make sure to expand it along the most efficient row/column.

$$\begin{vmatrix} 4 & 0 & -7 & 3 & -5 \\ 0 & 0 & 2 & 0 & 0 \\ 7 & 3 & -6 & 4 & -8 \\ 5 & 0 & 5 & 2 & -3 \\ 0 & 0 & 9 & -1 & 2 \end{vmatrix}$$

3. True or false:
   (a) $\det(A+B) = \det A + \det B$
   (b) If two columns of a matrix are equal, then its determinant is 0.
   (c) If $\det A = 0$, then two rows or two columns of A are the same, or one row or one column of A is zero.
   (d) $\det A^T = (-1)^n \det A$.

4. Find a formula for $\det(rA)$ in terms of r and $\det A$.

5. Use Cramer's rule to solve the following system of equations.

$$\begin{aligned} 3x_1 - 2x_2 &= 3 \\ -4x_1 + 6x_2 &= -5 \end{aligned}$$

6. Find the values for the parameter s that make the following system of equations have a unique solution. Then use Cramer's rule to find the solution (the solution should depend on s).

$$\begin{aligned} 3sx_1 + 5x_2 &= 3 \\ 12x_1 + 5sx_2 &= 2 \end{aligned}$$

7. Use Cramer's rule to find the inverse of the matrix:

$$\begin{bmatrix} 0 & -2 & -1 \\ 5 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

8. Find the volume of the parallelepiped with one vertex at the origin and adjacent vertices at $(1, 0, -3)$, $(1, 2, 4)$, and $(5, 1, 0)$.

9. Is $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ an eigenvector of $\begin{bmatrix} 3 & 6 & 7 \\ 3 & 3 & 7 \\ 5 & 6 & 5 \end{bmatrix}$? If so, find the eigenvalue.

   Is $\lambda = -2$ an eigenvalue of $\begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$? Why or why not?

10. Find all the eigenvalues and the eigenvectors of the following matrices.

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 5 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 5 \\ 0 & 0 & -1 \end{bmatrix}$$

11. Without writing the matrix, find one eigenvalue and eigenvector of the following linear transformations

    T is the transformation on $\mathbb{R}^2$ that reflects points across some line through the origin.

    T is the transformation on $\mathbb{R}^3$ that rotates points about some line through the origin.

12. Construct an example of a 2×2 matrix with only one distinct eigenvalue.

13. Consider an $n \times n$ matrix A with the property that the row sums all equal the same number s. Show that s is an eigenvalue of A. [*Hint:* Find an eigenvector.]

14. Let λ be an eigenvalue of an invertible matrix A. Show that λ^{-1} is an eigenvalue of A^{-1}. [*Hint:* Suppose a nonzero $\mathbf{x}$ satisfies $A\mathbf{x} = \lambda\mathbf{x}$.]