Lecture 15:

\( P(\cdot | F) \) is a probability.

Recall the axioms of probability.

1. \( P(\epsilon) \geq 0 \)
2. \( P(S) = 1 \)
3. If \( \epsilon_1, \epsilon_2, \ldots \) mutually exclusive,
   then \( P(\bigcup_i \epsilon_i) = \sum_{i=1}^\infty P(\epsilon_i) \)

The same things are true for \( P(\cdot | F) \).

\begin{align*}
(1) \quad P(\epsilon_1 | F) &= \frac{P(\epsilon_1 F)}{P(F)} \leq 1 \quad \text{since} \quad P(\epsilon_1 F) \leq P(F) \quad \text{since} \quad \epsilon_1 \subseteq F \subseteq F \\
(2) \quad P(S | F) &= \frac{P(S F)}{P(F)} = \frac{P(F)}{P(F)} = 1 \\
(3) \quad \epsilon_1, \epsilon_2, \ldots \text{ mutually exclusive.} \\
P(\bigcup_i \epsilon_i | F) &= \frac{P(\bigcup_i \epsilon_i F)}{P(F)} = \frac{P(\epsilon_1 F \bigcup \epsilon_2 F \bigcup \cdots)}{P(F)} = \sum_{i=1}^\infty \frac{P(\epsilon_i F)}{P(F)} \end{align*}
Example: Back to the example about accident-prone people.

A = "person is accident-prone"
R = "person is accident-prone"

\[ P(A|R) = 0.4 \quad P(R) = 0.3 \]
\[ P(A|R^c) = 0.1 \]

Q: What is the probability that someone who had an accident the first year has an accident in the second year?

We need to figure out how likely a person who has an accident is to be accident-prone.

\[ P(R|A) = \frac{P(A|R) \cdot P(R)}{P(A)} = \frac{0.4 \cdot 0.3}{P(A|R) \cdot P(R) + P(A|R^c) \cdot P(R^c)} = \frac{12}{0.19}. \]

Then \[ P(R^c|A) = 1 - P(R|A) = \frac{7}{19}. \]

\[ P(A_2|A_1) = P(A|R) \cdot P(R|A) + P(A|R^c) \cdot P(R^c|A) = 0.4 \cdot \frac{12}{19} + 0.1 \cdot \frac{7}{19} = \frac{5.5}{19} \approx 28\% . \]