Continuous random variables:

- Recall that a random variable $X$ was an object whose value was random in some way.
- A discrete random variable could only take finitely or countably infinitely many values.
  - i.e., $P(X = x) > 0$ for only $x = x_1, x_2, x_3, \ldots$
  - and $\sum_{i=1}^{\infty} P(X = x_i) = 1$.
- We had probability mass function of $X$
  - $p(x_i) = P(X = x_i)$

Now we'll do the same for random variables that can take infinitely many values in a continuum.

- $f(x)$ probability density function for $X$
- $P(X \in [a, b]) = \int_{a}^{b} f(x) \, dx$
- In general, $P(X \in B) = \int_{B} f(x) \, dx$. 

Of course, we should have:

\[ 1 = \mathbb{P}(X \in (-\infty, \infty)) = \int_{-\infty}^{\infty} f(x) \, dx = 1. \]

And

\[ \mathbb{P}(X = a) = \int_{a}^{a} f(x) \, dx = 0. \]

So the probability of \( X \) taking a specific value is always \( 0 \) for a continuous random variable \( X \).

**Example**: Amount of hours that a hard drive functions before breaking is a continuous r.v. with probability density function:

\[ f(x) = \begin{cases} \frac{1}{100} e^{-x/100} & x \geq 0 \\ 0 & x < 0 \end{cases} \]

1) What is \( C \)?
2) \( \mathbb{P}(X \in [50, 150]) = ? \)
3) \( \mathbb{P}(X < 100) = ? \)

**Solution**: We must have \( \int_{-\infty}^{\infty} f(x) \, dx = 1 \)

So

\[ \int_{-\infty}^{0} 0 \, dx + \int_{0}^{\infty} \frac{1}{100} e^{-x/100} \, dx = 1 \]

\[ 1 = C \left[ e^{-x/100} \right]_{0}^{\infty} = C \cdot \left( -100 \right) e^{-\infty} \left|_{0}^{\infty} \right. = C \cdot (-100)(0-1) = 100C \] so \( C = \frac{1}{100} \).
2) \( P(X \in [50, 150]) = \frac{1}{100} \int_{50}^{150} e^{-\frac{x}{100}} \, dx \)

\[
= \frac{1}{100} \left[ -e^{-\frac{x}{100}} \right]_{50}^{150}
\]

\[
= -e^{-\frac{150}{100}} - (-e^{-\frac{50}{100}})
\]

\[
= e^{-1.5} - e^{-0.5} \approx 0.38
\]

3) Similarly,

\( P(X < 100) = \int_{0}^{100} e^{-\frac{x}{100}} \, dx = -e^{-x/100} \bigg|_{0}^{100} = 1 - e^{-1} \approx 0.63 \)

**Definition:** The cumulative distribution function of \( X \) is defined as:

\[ F(a) = P(X \leq a) = \int_{-\infty}^{a} f(x) \, dx. \]

Note that \( f \) and \( F \) can be obtained from each other:

\[ \frac{d}{da} F(a) = f(a) \]

(So it's just a different way of expressing the same information.)

**Question:**

1) If \( X \) has density function \( f(x) \), what is the density function of \( Y = 2X \)?

2) If \( X \) has distribution function \( F(x) \), what is the distribution function of \( Y = 2X \)?