Projects: Tumor Growth
You may choose either one.

1. Continuum model. Take the model of tumor growth given by

\[ D \nabla^2 \sigma - \Gamma = 0 \]

\[ \nabla \cdot \mathbf{v} = \lambda_p \sigma - \lambda_A \]

\[ \mathbf{v} = -\mu \nabla p \]

where \( \sigma \) is the concentration of a vital nutrient and \( \mathbf{v} \) is the cell-velocity. Take the boundary conditions \( \sigma = \sigma_\infty, p = \kappa \tau \) on the tumor/host interface \( \Sigma(t) \) where \( \kappa \) is the curvature and boundedness conditions \( \partial \sigma / \partial r = \partial p / \partial r = 0 \) at \( r = 0 \). Assume the normal velocity \( V \) of \( \Sigma(t) \) is given by

\[ V = -\mu \nabla p \cdot \mathbf{n} \]

Where \( \mathbf{n} \) is the outward unit normal (pointing into the host).

(a). Nondimensionalize the system.

(b). Determine the steady-states. That is, determine under what conditions steady-states exist.

(c). Analyze the stability of the steady-states.

2. Discrete tumor growth model.

Let \( x_j(t) \) denote the position of a \( j^{th} \) tumor cell at time \( t \). Let us suppose that they interact according to a general potential energy:

\[ E(t) = \sum_{i,j \text{ and } i \neq j} C_A e^{-|x_i - x_j|/l_A} - C_R e^{-|x_i - x_j|/l_R} \]

where \( C_A, l_A \) and \( C_R, l_R \) are constants. Suppose the evolution is given by:

\[ \frac{dx_j}{dt} = \frac{\partial}{\partial x_j} \sum_i C_A e^{-|x_i - x_j|/l_A} - C_R e^{-|x_i - x_j|/l_R} . \]

This assumes tumor cells interact with one another mechanically.
(a). Demonstrate the evolution always ensures that $E(t)$ is non-increasing in time.

(b). Non-dimensionalize the system. What are the nondimensional parameters?

(c). Find the equilibrium solution for 2 tumor cells. Is it stable? Why?

(d). Solve the system numerically for 50 tumor cells. What is the result of the evolution?

(e). Extra credit. Devise a scheme by which tumor cells undergo mitosis. Then, what happens to the system?