1. Suppose 13 points in the plane are given, numbered $P_1, P_2, \ldots P_{13}$. Consider the 13 line segments $P_1P_2, P_2P_3, \ldots P_{13}P_1$. Is it possible that there is a straight line that crosses each segment (crossing means intersecting the segment in one point, not an endpoint)?

2. If $A$, $B$, and $C$ are the 3 angles of a triangle, show that
\[
\frac{\sin(A) + \sin(B)}{2} \leq \sin\left(\frac{A + B}{2}\right).
\]

3. Show that
\[
1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \cdots + \frac{1}{\sqrt{n}} \leq 2\sqrt{n}.
\]

4. Find all real numbers $x$ that satisfy
\[
\frac{3}{x - 1} < \frac{2}{x + 1}.
\]

5. Suppose $a$ is a real number and $a + 1/a$ is rational. Prove that $a^n + 1/a^n$ is rational for every integer $n$. 