1. Show that
\[ \sum_{k=0}^{n} \frac{1}{k+1} \binom{n}{k} = \frac{2^{n+1} - 1}{n+1}. \]

2. Suppose \( x_1, x_2, \ldots \) is a sequence of positive real numbers satisfying \( x_{n+1} \leq x_n + 1/n^2 \) for all \( n \geq 1 \). Prove that \( \lim_{n \to \infty} x_n \) exists.

3. Show that
\[ F_1 + F_2 + \cdots + F_n = F_{n+2} - 1 \]
where \( F_n \) is the \( n \)-th Fibonacci number.

4. Evaluate
\[ S_1 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots \]

5. Express \( \prod_{n=0}^{\infty} (1 + x^{2n}) \) as a rational function of \( x \) (a quotient of polynomials).