Practice Exam, Math 194

Rules: find a quiet spot where you can work on these problems uninterrupted for 80 minutes. Write your solutions carefully and completely. **Work by yourself.** No books, notes, calculators, internet, or other help allowed.

Your performance on this practice test will not affect your final grade (pass/no pass) in Math 194.

Do as many problems as you can, and return your completed exam by 7pm on Tuesday, November 14. You can either bring it to class, or leave it under the door of RH 510L, or email it to `krubin@uci.edu`. If you don't make progress on any of the problems, there is no need to turn anything in.

1. Suppose S is a set with 10 elements. How many subsets of S have an odd number of elements?

2. For every positive integer k, let $f_1(k)$ denote the sum of the squares of the base 10 digits of k. For $n \geq 2$ let $f_n(k) = f_1(f_{n-1}(k))$. Find $f_{2011}(11)$.

3. Evaluate the sum
$$\sum_{n=1}^{1{,}000{,}000} \frac{1}{\langle \sqrt{n} \rangle}$$
where $\langle x \rangle$ denotes the integer *closest* to x.

4. Show (without using a calculator or doing extensive computation) that
$$\log_{2011} 2012 + \log_{2012} 2011 > 2$$
($\log_a b$ denotes the logarithm base a of b).

5. Let $a_1, a_2, \ldots, a_{65}$ be positive integers, none of which has a prime factor greater than 13. Prove that, for some i, j with $i \neq j$, the product $a_i a_j$ is a perfect square.

6. For every $n \geq 1$, let
$$x_n = \sqrt{3 + \sqrt{3 + \sqrt{3 + \cdots \sqrt{3}}}}$$
with n 3's. Show that $\lim_{n\to\infty} x_n$ exists, and find its value.