(1) What is the remainder when the polynomial \( f(x) \) is divided by \((x - a)^2\)? by \(x^2 - a\)?

(2) For which real values of \( p \) and \( q \) are the roots of the polynomial \( x^3 - px^2 + 11x - q \) three successive integers? Give the roots in these cases.

(3) Find, without a calculator, the smallest integer larger than \((\sqrt{3} + \sqrt{2})^6\).

(4) (a) Determine all \((a, b, c)\) such that the quadratic polynomial, \( Q(n) = an^2 + bn + c \), assumes integer values for every integer \( n \).

(b) Determine all \((a, b, c, d)\) such that the cubic polynomial, \( C(n) = an^3 + bn^2 + cn + d \), assumes integer values for every integer \( n \).

(5) If \( P_n(x) \) denotes a polynomial of degree \( n \) such that \( P_n(k) = \frac{1}{k} \) for \( k = 1, 2, 3, \ldots, n + 1 \), determine \( P_n(n + 2) \).

(6) For which real numbers \( c \) is there a straight line that intersects the curve

\[
y = x^4 + 9x^3 + cx^2 + 9x + 4
\]

in 4 distinct points? (Putnam 1994)

(7) Let \( k \) be a positive integer. Find all polynomials \( P(x) \) with coefficients that are real numbers, satisfying the identity

\[
P(P(x)) = P(x)^k.
\]

(8) For which ordered pairs of real numbers \((b, c)\) do both of the roots of the quadratic polynomial \( z^2 + bz + c \) lie inside the unit disk \( \{ z : |z| < 1 \} \)? (Putnam 1975)

(9) Let \( f(x) \) be a polynomial, and \( a \neq b \). Suppose \( f(x) \) leaves the remainder \( A \) when divided by \( x - a \) and the remainder \( B \) when divided by \( x - b \). Find the remainder when \( f(x) \) is divided by \((x - a)(x - b)\).

(10) Is there an infinite sequence of nonzero real numbers \( a_0, a_1, a_2, \ldots \) such that the polynomial \( a_0 + a_1x + a_2x^2 + \cdots + a_nx^n \) has exactly \( n \) real roots?