Example (Martin-McMillen, 1998). *The elliptic curve*

$$y^2+xy+y=x^3-19252966408674012828065964616418441723x$$
$$+326855007277163762579233470714520442959074430563456140066$$

has rank at least 23.

Proof.

$$P_1=\Big(\frac{169021360446217242755846613925950}{119224493521},$$
$$-\frac{694555197849719936798075523086097394308582488120}{41166906143372569}\Big),$$
$$P_2=\Big(\frac{66478822724661038216347720465710}{30891226081},$$
$$-\frac{171370238447109871400493873099459538929462135440}{5429411004770479}\Big),$$

$\dots$

How do we know rank$(E_{s(f(t))})\geq 3$? Let

$$x_1(t)=12(t^4-4t^3+6t^2-4t+2)^2(t^4-4t^3+12t^2-16t+8),$$
$$y_1(t)=36t(t-2)(t^4-4t^3+6t^2-4t+2)^2(t^4-4t^3+12t^2-16t+8)^2,$$
$$x_2(t)=\frac{(t^2-4t+2)(t^2-2)(t^4-4t^3+6t^2-4t+2)^2(t^4-4t^3+12t^2-16t+8)}{36(t-1)^2},$$
$$y_2(t)=\frac{(t^2-4t+2)^2(t^2-2)^2(t^4-4t^3+6t^2-4t+2)^2(t^4-4t^3+12t^2-16t+8)^2}{216(t-1)^3},$$
$$x_3(t)=12(t^2-4t+2)(t^2-2)(t^4-4t^3+6t^2-4t+2)^2,$$
$$y_3(t)=36(t^2-4t+2)^2(t^2-2t+2)(t^2-2)^2(t^4-4t^3+6t^2-4t+2)^2.$$

Then

$$(x_1(t),y_1(t)),\ (x_1(t),y_1(t)),\ (x_1(t),y_1(t))\ \in\ E_{f(t)}(\mathbb{Q})$$

for every t, and for all but finitely many t these points are independent.