Math 230C, problem set #3

due April 29

(1) Write down the character table of A_4 (i.e., list all irreducible characters and their values on all conjugacy classes).

(2) Write down the character table of D_{10}.

(3) If R is a commutative ring, $I \subset R$ is an ideal, and M is an R-module, show that $M \otimes_R (R/I) \cong M/IM$.

(4) Show that $(\mathbf{Z}/n\mathbf{Z}) \otimes_{\mathbf{Z}} (\mathbf{Z}/m\mathbf{Z}) \cong \mathbf{Z}/(m,n)\mathbf{Z}$.

(5) Suppose R is a commutative integral domain, F is the field of fractions of R, and M is an R-module. Show that every element of $F \otimes_R M$ is of the form $1/d \otimes m$ with $d \in R$ and $m \in M$.

(6) Show that $\mathbf{Z}[i] \otimes_{\mathbf{Z}} \mathbf{R} \cong \mathbf{C}$ as rings.

(7) Suppose R is a ring and S is an R-algebra (i.e., there is a ring homomorphism $R \to S$). Show that $S \otimes_R R[x] \cong S[x]$ as rings. If I is an ideal of R, show that $S \otimes_R (R/I)[x] \cong (S/IS)[x]$ as rings.

(8) Show that the principal ideal (5) is a prime ideal in $\mathbf{Z}[\sqrt{7}]$. (Hint: show first that $\mathbf{Z}[\sqrt{7}] \cong \mathbf{Z}[x]/(x^2-7)$. Use #3 to show that $(\mathbf{Z}[x]/(x^2-7))/(5) \cong (\mathbf{Z}[x]/(x^2-7)) \otimes_{\mathbf{Z}} (\mathbf{Z}/5\mathbf{Z}) \cong (\mathbf{Z}/5\mathbf{Z})[x]/(x^2-7)$.)