Math 230C, problem set #4

due May 13

(1) Rotman #9.70

(2) Rotman #9.86

(3) Rotman #9.89

(4) Rotman #9.90

(5) Rotman #9.95

(6) Suppose that R is a commutative ring and M is a cyclic R-module.
   (a) Show that the tensor algebra $T(M)$ is isomorphic to the symmetric algebra $\mathrm{Sym}(M)$.
   (b) Show that the exterior algebra $\bigwedge(M)$ is isomorphic to $R \oplus M$ (with what multiplication?).

(7) Suppose R is a (commutative) integral domain, and $I \subset R$ is an ideal.
   (a) Show that $\wedge^2(I)$ is a torsion R-module.
   (b) Let $R = \mathbf{Z}[x, y]$ and $I = (x, y)$. Show that $\wedge^2(I) \neq 0$. Hint: show that the map
   $$(ax + by, cx + dy) \mapsto ad - bc$$
   from $I \times I$ to R/I is R-bilinear, and use this to construct a nonzero map from $\wedge^2(I)$.