

Math 124, problem set #5

due May 16, 2006

- (1) Describe the splitting field of $x^4 - 4$. What is its degree over \mathbf{Q} ?
- (2) Let $\mathbf{E} = \mathbf{Q}(\sqrt{3}, i)$ (where $i = \sqrt{-1}$). What is $G(\mathbf{E}/\mathbf{Q})$? (Hint: what is $G(\mathbf{E}/\mathbf{Q}(\sqrt{3}))$? What is $G(\mathbf{E}/\mathbf{Q}(i))$? How big can $G(\mathbf{E}/\mathbf{Q})$ be?)
- (3) Suppose that H and K are two *different* finite groups of automorphisms of a field \mathbf{E} . Show that the fixed fields $E^H \neq E^K$.
- (4) Show that a regular 65537-gon is constructible. You may use the fact that 3 has order 65536 = 2^{16} modulo 65537.
- (5) Suppose a is algebraic over \mathbf{Q} , and let $d = [\mathbf{Q}(a) : \mathbf{Q}]$. What are all the possibilities for $[\mathbf{Q}(a^4) : \mathbf{Q}]$ (in terms of d)? Illustrate your answer with concrete examples.