Math 124, problem set #6

due May 23, 2006

(1) Find all fields $\mathbf{K}$ such that $\mathbf{Q} \subseteq \mathbf{K} \subseteq \mathbf{Q}(e^{2\pi i/5})$. Justify your answer.

(2) Suppose $\mathbf{E}$ is a normal extension of $\mathbf{F}$, and $\mathbf{F} \subseteq \mathbf{K} \subseteq \mathbf{E}$. Show that if $\phi \in G(\mathbf{K}/\mathbf{F})$, then ϕ can be extended to an automorphism of $\mathbf{E}$ (i.e., there is an automorphism ψ of $\mathbf{E}$ whose restriction to $\mathbf{K}$ is ϕ).

(3) Let $\mathbf{E} = \mathbf{Q}(\sqrt[6]{5})$. What is $G(\mathbf{E}/\mathbf{Q})$? What is $\mathbf{E}^{G(\mathbf{E}/\mathbf{Q})}$?

(4) Show that if $\mathbf{K}/\mathbf{F}$ is a finite extension, then there is a normal extension $\mathbf{E}$ of $\mathbf{F}$ that contains $\mathbf{K}$.

(5) Use problem (4) to show that if $\mathbf{E}/\mathbf{F}$ is a finite extension (not necessarily normal) then there are only finitely many fields $\mathbf{K}$ such that $\mathbf{F} \subset \mathbf{K} \subset \mathbf{E}$.