(1) Show that if $G$ is a finite commutative group, then $G$ is solvable.

(2) Show that $D_{2n}$, the dihedral group of order $2n$, is solvable.

(3) Show that if $G$ is a solvable group, and $H$ is a subgroup of $G$, then $H$ is solvable.

(4) Suppose $F$ is a field, $f(x) \in F[x]$ is an irreducible polynomial, and $r_1, r_2$ are roots of $f$. Show that the fields $F(r_1)$ and $F(r_2)$ are isomorphic (i.e., there is a bijection $\phi : F(r_1) \rightarrow F(r_2)$ that preserves addition and multiplication).

(5) What is wrong with the following proof of the (false) statement that every normal extension $E/F$ is solvable?

"Proof". We proceed by induction on $[E:F]$. If $[E:F] = 1$, then $E = F$ and $E/F$ is solvable.

Suppose $[E:F] > 1$, and let $p$ be a prime dividing $[E:F]$. Then $G(E/F)$ has an element $\phi$ of order $p$. Let $H$ be the subgroup of order $p$ generated by $\phi$, and let $K = E^H$. Then $E/K$ is normal of prime order $p$. Since $[K:F] < [E:F]$, by induction we know that $K/F$ is solvable. Therefore we have

$$F = F_0 \subset F_1 \subset \cdots \subset F_N = K$$

with each $F_{i+1}/F_i$ normal of prime degree. Now the tower

$$F = F_0 \subset F_1 \subset \cdots \subset F_N = K \subset F_{N+1} = E$$

shows that $E/F$ is solvable.