Math 180, problem set #7
due March 6, 2007

(1) Compute the continued fraction for $105/38$. Using this, find integers x and y such that $105x + 38y = 1$.

(2) Compute the continued fraction for $95/43$. Using this, find integers x and y such that $95x + 43y = 1$.

(3) Compute the continued fraction for $\sqrt{5}$. Find the first convergent of this continued fraction that approximates $\sqrt{5}$ to within $1/10^5$.

(4) Compute the continued fraction for $\sqrt{6}$. Find the first convergent of this continued fraction that approximates $\sqrt{6}$ to within $1/10^5$.

(5) Evaluate the infinite continued fraction $[2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, \ldots]$.

(6) Using a calculator, compute the beginning of the continued fraction for $\sqrt[3]{2}$. Find the first convergent of this continued fraction that approximates $\sqrt[3]{2}$ to within $1/10^5$.

(7) Show that if p, q are relatively prime positive integers, d is an integer that is not a square, and
$$\left| \frac{p}{q} - \sqrt{d} \right| < \frac{1}{(\sqrt{d} + 1)q^2}$$
then $p^2 - dq^2 = \pm 1$. (Hint: show that $|p^2 - dq^2| < 2$.)

(8) Using problem (7) and problem (3), find two pairs (x, y) of positive solutions of the equation $x^2 - 5y^2 = 1$.

(9) Using problem (7) and problem (4), find two pairs (x, y) of positive solutions of the equation $x^2 - 6y^2 = 1$.