Math 194, problem set #2

For discussion Tuesday October 16

1. Show that if the fraction a/b is expressed as a decimal number (where a, b are positive integers), it either terminates, or begins repeating after most $b-1$ decimal places. (Hint: if you actually work out the long division, dividing a by b, what does it mean for the decimal expansion to repeat?)

2. The Fibonacci numbers are defined by the recurrence relationship

$$F_1 = 1 \qquad F_2 = 1 \qquad F_{n+2} = F_{n+1} + F_n \quad \text{for} \quad n = 1, 2, 3, \ldots$$

   Show

$$F_1^2 + F_2^2 + \cdots + F_n^2 = F_n F_{n+1}$$

3. Inside a unit square, 101 points are placed. Show that some three of them form a triangle with area no more than .01.

4. Show that for $n \geq 6$ a square can be dissected into n smaller squares, not necessarily all of the same size.

5. Suppose 9 integer lattice points are given in $\mathbf{R}^3$. Show that there is at least one pair of these points such that the line segment joining them contains an additional integer lattice point.

6. Consider five points in the interior of a square of side length 1. Show that there is at least one pair of points a distance of at most $1/\sqrt{2}$ apart. Show that $1/\sqrt{2}$ is best possible.

7. Let S denote an $n \times n$ lattice square, $n \geq 3$. Show that it is possible to draw a polygonal path consisting of $2n-2$ segments which will pass through all of the n^2 lattice points of S.

8. In how many ways can a $2 \times n$ square be tiled with 2×1 dominos?

9. Show that in any group of 6 people there are either 3 mutual acquaintances or 3 mutual strangers.

10. The numbers from 1 to 10 are arranged in some order around a circle. Show that there are three consecutive numbers whose sum is at least 17.