

## Math 194, problem set #3

For discussion Tuesday October 23

- (1) Prove that  $36^{36} + 41^{41}$  is divisible by 77.
- (2) A positive integer's digits are all 6 or 0; can it be a perfect square?
- (3) Show that  $x^2 - y^2 = a^3$  always has positive integer solutions for  $x$  and  $y$  whenever  $a$  is an integer greater than one. For which values of  $a$  is the solution unique?
- (4) What is the smallest natural number that leaves remainders 1, 2, 3, 4, 5, 6, 7, 8 and 9 when divided by 2, 3, 4, 5, 6, 7, 8, 9 and 10, respectively?
- (5) Determine all  $n$  such that the  $n$ -digit number  $R_n = 1111 \cdots 111$  is divisible by 37. For which  $n$  is it divisible by 41?
- (6) Show that there is a sequence of  $10^6$  consecutive positive integers, each of which is divisible by the cube of some integer greater than 1.
- (7) (a) How many zeroes does  $100!$  end in?  
(b) What is the final non-zero digit in  $100!$  ?
- (8) Let  $A = 4444^{4444}$ . Let  $B$  be the sum of the (base 10) digits of  $A$ . Let  $C$  be the sum of the digits of  $B$ . What is the sum of the digits of  $C$ ?
- (9) Prove that the sequence (in base-10 notation)  
 $11, 111, 1111, 11111, \dots$   
contains no squares.