(1) Suppose $K \subset F$, $K \subset L$ are number fields, and $p$ is a prime of $K$.
   (a) Show that if $p$ splits completely in $F$ and $L$, then $p$ splits completely in $FL$.
   (b) Show that $p$ splits completely in $F$ if and only if $p$ splits completely in the Galois closure of $F/K$.

(2) Suppose $G$ is a finite abelian group, and let $\hat{G} = \{\chi : \chi : G \to \mathbb{C}^\times\}$.
   (a) Show that if $g \in G$ then
      $$\sum_{\chi \in \hat{G}} \chi(g) = \begin{cases} 0 & \text{if } g \neq 1 \\ |G| & \text{if } g = 1. \end{cases}$$
   (b) Show that if $\chi \in \hat{G}$ then
      $$\sum_{g \in G} \chi(g) = \begin{cases} 0 & \text{if } \chi \neq 1 \\ |G| & \text{if } \chi = 1. \end{cases}$$

(3) (a) Describe as best you can the rational primes that split completely in $\mathbb{Q}(\sqrt[3]{2})$.
    (b) Describe as best you can the rational primes that split completely in $\mathbb{Q}(\sqrt[3]{2}, e^{2\pi i/3})$.
    (c) Describe as best you can the primes of $\mathbb{Q}(e^{2\pi i/3})$ that split completely in $\mathbb{Q}(\sqrt[3]{2}, e^{2\pi i/3})$.
    (d) How are these three sets of primes related?

(4) In class we used the fact that for any number field $K$ and cycle $\epsilon$, there is a finite abelian extension $F/K$ such that $N_{F/K}I_F(\epsilon) \subset \varphi_{\epsilon}$. Prove this when $K = \mathbb{Q}$ and $\epsilon = p\infty$. 