Math 232B
Problem set #4, due February 20, 2008

(1) Suppose $F/K$ is a finite extension of local fields. Prove that $N_{F/K} F^\times$ and $N_{F/K} U_F$ are both open and closed in $K^\times$.

(2) Using the correspondence between quadratic extensions of $\mathbb{Q}_p$ and subgroups of index 2 in $\mathbb{Q}_p^\times$ given by local class field theory, match up the extensions with the corresponding subgroups that you found in problems 1 and 2 of problem set 3.

(3) Suppose $R$ is an integral domain, and let $\mathfrak{M} = XR[[X]]$. Suppose $f \in \mathfrak{M}$.
   (a) Show that there is a $g \in \mathfrak{M}$ with $f \circ g = X$ (i.e., $f(g(X)) = X$) if and only if $f'(0) \in R^\times$.
   (b) Show that if $g \in \mathfrak{M}$ and $f \circ g = X$, then $g \circ f = X$.

(4) Suppose $R$ is a $\mathbb{Q}$-algebra (for example, a field of characteristic zero) and $F$ is a formal group over $R$. Show that the isomorphism $\lambda(X) \in R[[X]]$ from $F$ the additive formal group such that $\lambda(X) \equiv X \pmod{\operatorname{deg} 2}$ is unique (existence proved in class). Hint: show

$$\lambda'(X) = \left(\frac{\partial}{\partial Y} F(X, Y)|_{Y=0}\right)^{-1}.$$