Math 232C
Problem set #3, due April 23, 2008

In all of these problems, suppose $G$ is a finite group.

(1) Suppose $A$ is an induced $G$-module. Show that $\hat{H}^{-1}(G, A) = 0$.

(2) Suppose $A$ is a $G$-module and $H$ is a subgroup of $G$. Show that the corestriction map

$$\text{Cor} : \hat{H}^0(H, A) \to \hat{H}^0(G, A)$$

defined by dimension-shifting the natural map $\text{Cor} : \hat{H}^{-1}(H, A) \to \hat{H}^{-1}(G, A)$, is the same as the map induced by multiplication by $\sum_{g \in G/H} g$ on $A^H$. (You will need to use the explicit definition of dimension-shifting.)

(3) Suppose that $A$ is a $|G|$-module ($A$ is not necessarily finite), and let $f : A \to A$ be multiplication by $|G|$.

(a) If $f$ is an isomorphism, show that $\hat{H}^q(G, A) = 0$ for every $A$.

(b) Give an example of $G$ and $A$ such that $f$ is injective, but not all $\hat{H}^q(G, A)$ are zero.

(c) Give an example of $G$ and $A$ such that $f$ is surjective, but not all $\hat{H}^q(G, A)$ are zero.

(4) Show that if $n \in \mathbb{Z}^+$ and $F/K$ is a finite Galois extension of degree prime to $n$, then the map $K^\times/(K^\times)^n \to F^\times/(F^\times)^n$ is injective.

(5) Suppose $G$ is cyclic. Compute the Herbrand quotient for each of the modules

(a) $\mathbb{Z}$ (with trivial action)

(b) $\mathbb{Q}/\mathbb{Z}$ (with trivial action)

(c) the ideal $I_G \subset \mathbb{Z}[G]$

You should be able to do (b) and (c) with minimal effort, after doing (a).