Math 232C

Problem set #3, due April 23, 2008

In all of these problems, suppose G is a finite group.

(1) Suppose A is an induced G-module. Show that $\hat{H}^{-1}(G, A) = 0$.

(2) Suppose A is a G-module and H is a subgroup of G. Show that the corestriction map

$$\mathrm{Cor} : \hat{H}^0(H, A) \to \hat{H}^0(G, A)$$

defined by dimension-shifting the natural map $\mathrm{Cor} : \hat{H}^{-1}(H, A) \to \hat{H}^{-1}(G, A)$, is the same as the map induced by multiplication by $\sum_{g \in G/H} g$ on A^H. (You will need to use the explicit definition of dimension-shifting.)

(3) Suppose that A is a $|G|$-module (A is not necessarily finite), and let $f : A \to A$ be multiplication by $|G|$.

  (a) If f is an isomorphism, show that $\hat{H}^q(G, A) = 0$ for every A.

  (b) Give an example of G and A such that f is injective, but not all $\hat{H}^q(G, A)$ are zero.

  (c) Give an example of G and A such that f is surjective, but not all $\hat{H}^q(G, A)$ are zero.

(4) Show that if $n \in \mathbf{Z}^+$ and F/K is a finite Galois extension of degree prime to n, then the map $K^\times/(K^\times)^n \to F^\times/(F^\times)^n$ is injective.

(5) Suppose G is cyclic. Compute the Herbrand quotient for each of the modules

  (a) $\mathbf{Z}$ (with trivial action)

  (b) $\mathbf{Q}/\mathbf{Z}$ (with trivial action)

  (c) the ideal $I_G \subset \mathbf{Z}[G]$

You should be able to do (b) and (c) with minimal effort, after doing (a).