# Math 232C

Problem set #6, to discuss on May 19, 2008

Fix a rational prime $p$, and for every prime $\ell$ let $A_\ell$ be the subgroup of $\overline{\mathbf{Q}_p}^\times$ generated by a primitive $\ell$-th root of unity $\zeta_\ell$ and $p^{1/\ell}$. Let $G = \mathrm{Gal}(\overline{\mathbf{Q}_p}/\mathbf{Q}_p)$.

(1) Show that $A_\ell \cong (\mathbf{Z}/\ell\mathbf{Z})^2$ as abelian groups.

(2) Show that $A_\ell^* \cong A_\ell$ as $G$-modules (recall that $A_\ell^* = \mathrm{Hom}(A_\ell, \boldsymbol{\mu}_\infty)$).

(3) Show that there is an exact sequence of $G$-modules

$$0 \longrightarrow \boldsymbol{\mu}_\ell \longrightarrow A_\ell \longrightarrow \mathbf{Z}/\ell\mathbf{Z} \longrightarrow 0.$$

(4) Compute $H^i(G, A_\ell)$ for $i \geq 0$ (use Tate duality for $H^2$).

(5) If $\ell \neq p$ show that $A_\ell$ is unramified, and compute $H^1_{\mathrm{ur}}(G, A_\ell)$.