SOUTHERN CALIFORNIA NUMBER THEORY DAY
OCTOBER 12, 2019

ABSTRACTS

Jim Brown, *Congruence primes for automorphic forms on unitary groups and applications to the arithmetic of Ikeda lifts*

It is well-established that studying congruences between automorphic forms can be used to provide interesting arithmetic results. In addition to these applications, classifying congruences between automorphic forms is an interesting problem in its own right. In this talk I'll survey a few known results before moving on to discussing some recent results with Kris Klosin in which we provide a sufficient condition for a prime to be a congruence prime for an automorphic form f on the unitary group $\mathrm{U}(n,n)(\mathbb{A}_F)$ for a large class of totally real fields F via a divisibility of a special value of the standard L-function associated to f. If there is time, I will also discuss some p-adic properties of the Fourier coefficients of an Ikeda lift I_ϕ (of an elliptic modular form ϕ) on $\mathrm{U}(n,n)(\mathbb{A}_\mathbb{Q})$ proving that they are p-adic integers which do not all vanish modulo p. In this case we show the condition of being a congruence prime for I_ϕ is controlled by the p-divisibility of a product of special values of the symmetric square L-function of ϕ.

Arul Shankar, *Squarefree sieves in arithmetic statistics*

A classical question in analytic number theory is: given a polynomial with integer coefficients, how often does it take squarefree values? In arithmetic statistics, we are particularly interested in the case of discriminant polynomials. In this talk, I will present several different cases of this question. First, we will consider a classical result of Davenport–Heilbronn which considers the case of discriminants of binary cubic forms. Then, I will discuss joint work with Bhargava in which we consider the case of discriminants of ternary cubic forms.

Third, I will describe joint and ongoing work with Bhargava and Wang, in which we consider different families of degree-n polynomials in one variable, and determine the proportion of those having squarefree discriminant. Finally, I will describe various applications of these results.

Ila Varma, *Armitage-Frohlich theorems and heuristics for narrow class groups of odd abelian number fields*

Armitage-Frohlich proved that the difference between the 2-rank of the narrow class group and of the class group of a number field with r_1 real embeddings is at most $r_1/2$. For instance, this implies that for cyclic cubic fields the difference is at most 1. However, the Galois module structure of class groups of cyclic cubic fields requires that the 2-rank is even, and so combining these two results implies that the 2-rank of the narrow class group of a cyclic cubic field is equal to the 2-rank of its class group! For abelian number fields of odd degree, we will discuss a generalization of Armitage-Frohlich's theorem that takes into consideration the Galois module structure of their class groups. After introducing the determined structure, we will describe how to make predictions in the vein of Cohen-Lenstra for the distribution of narrow class groups in families where the (odd) degree and (abelian) Galois group are both fixed. These results are joint work with Ben Breen and John Voight.

Bianca Viray, *Isolated points on modular curves*

Faltings' theorem on rational points on subvarieties of abelian varieties can be used to show that al but finitely many algebraic points on a curve arise in families parametrized by $\mathbb{P}^1$ or positive rank abelian varieties; we call these finitely many exceptions isolated points. We study how isolated points behave under morphisms and then specialize to the case of modular curves. We show that isolated points on $X_1(n)$ push down to isolated points on a modular curve whose level is bounded by a constant that depends only on the j-invariant of the isolated point. This is joint work with A. Bourdon, O. Ejder, Y. Liu, and F. Odumodu.