SOUTHERN CALIFORNIA NUMBER THEORY DAY, OCTOBER 25, 2008

ABSTRACTS

Jeff Achter, *The p-rank strata of the moduli space of curves*

Let E be an elliptic curve over an algebraically closed field k of characteristic $p > 0$. Then the physical p-torsion $E[p](k)$ is either trivial, and E is called supersingular, or $E[p](k)$ is a group of order p. More generally, if X/k is an abelian variety of dimension g, then $X[p](k)$ is isomorphic to $(\mathbf{Z}/p)^f$ for some number f, called the p-rank of X. The p-rank induces a stratification of the moduli space of abelian varieties; via the Torelli functor, it induces a stratification of the moduli space of (hyperelliptic) curves. I'll discuss recent results on the geometry of these strata, with special attention to their structure at the boundary of the moduli space. This information yields new applications about the prime-to-p part of the class group of a quadratic function field with specified geometric p-rank; the existence of absolutely simple hyperelliptic Jacobians of every p-rank; and the stratification of the moduli space of curves by Newton polygon.

(Joint work with Rachel Pries.)

Brian Conrad, *Finiteness theorems for algebraic groups*

Generalized ideal class groups can be described adelically in terms of a coset space for the group GL_1, and this in turn leads to a notion of "class number" (as the size of a certain set, if finite) for an arbitrary affine algebraic group over a global field. Related to this is the notion of the "Tate-Shafarevich set" of an algebraic group, which is tied up with questions relating global and local information. Finiteness of class numbers and Tate-Shafarevich sets for affine algebraic groups was proved by Borel and his coworkers over number fields, andif one grants the finiteness of Tate-Shafarevich groups for abelian varieties then Mazur showed how to get such finiteness for all algebraic group varieties over number fields (which has applications to the local-to-global principle for projective varieties over number fields).

The above methods do not apply over global function fields. After reviewing some history, I will explain the content of a recent classification theorem of "pseudo-reductive groups" proved jointly with Gabber and G. Prasad that makes it possible to prove the analogous finiteness theorems in the function field case away from characteristic 2. If time permits I will say something about how this classification theorem is used to get such results.

Jordan Ellenberg, *Stable cohomology of moduli spaces and Cohen-Lenstra conjectures over function fields*

A Hurwitz space $H_{G,n}$ is an algebraic variety parametrizing branched covers of the projective line with some fixed finite Galois group G. We will prove that, under some hypotheses on G, the rational i-th homology of the Hurwitz spaces stabilizes when the number of branch points is sufficiently large compared to$i$.

This purely topological theorem has some interesting number-theoretic consequences. It implies, for instance, a weak form of the Cohen-Lenstra conjectures over rational function fields, and some quantitative inverse Galois results over function fields. For instance, we show that the average size of the p-part of the class number of a hyperelliptic genus-g curve over $\mathbf{F}_q$ is bounded independently of g, when q is large enough relative to p; the key point here is q can be held fixed while g grows.

I will try to give a general overview of the dictionary between conjectures about topology of moduli spaces, on the one hand, and arithmetic distribution conjectures (Cohen-Lenstra, Bhargava, Malle, inverse Galois...) on the other.

(Joint work with Akshay Venkatesh and Craig Westerland.)

Cristian Popescu, 1-*motives and special values of equivariant L-functions*

We will discuss our recent proof (joint work with C. Greither) of a conjecture linking ℓ-adic realizations of 1-motives and special values of equivariant L-functions in characteristic p, refining earlier results of Deligne and Tate. As a consequence, we will give proofs (in the characteristic p setting) of various central classical conjectures on special values of L-functions, namely those due to Coates-Sinnott, Brumer-Stark, and Gross. Also, we will indicate how this theory can be extended to characteristic 0.