

SOUTHERN CALIFORNIA NUMBER THEORY DAY,
OCTOBER 18, 2014

ABSTRACTS

Amanda Folsom, *Quantum modular and mock modular forms.*

In 2010, Zagier defined the notion of a “quantum modular form,” and offered several diverse examples, including Kontsevich’s ‘strange’ function. Here, we construct infinite families of quantum modular forms, and prove one of Ramanujan’s remaining claims about mock theta functions in his last letter to Hardy as a special case of our work. We will show how quantum modular forms underlie new relationships between combinatorial mock modular and modular forms due to Dyson and Andrews-Garvan. This is joint work with Ken Ono (Emory U.) and Rob Rhoades (CCR-Princeton).

Daniel Kane, *On a problem related to the ABC conjecture.*

The *ABC* Conjecture, roughly stated says that the equation $A + B + C = 0$ has no solutions for relatively prime, highly divisible integers A , B , and C . If the divisibility criteria are relaxed, then solutions exist and a conjecture of Mazur predicts the density of such solutions. We discuss techniques for proving this conjecture for certain ranges of parameters.

Michelle Manes, *Some arithmetic properties of post-critically finite rational functions.*

Sarah Zerbes, *Euler systems and the Birch–Swinnerton-Dyer conjecture.*

The Birch–Swinnerton-Dyer conjecture is now a theorem, under some mild hypotheses, for elliptic curves over \mathbf{Q} with analytic rank ≤ 1 . One of the main ingredients in the proof is Kolyvagin’s theory of Euler systems: compatible families of cohomology classes which can be seen as an “arithmetic avatar” of an L-function. The existence of Euler systems in other settings would have similarly strong arithmetical applications, but only a small number of examples are known.

In this talk, I’ll introduce Euler systems and their uses, and I’ll describe the construction of a new Euler system, which is attached to the Rankin–Selberg convolution of two modular forms; this is joint work with Antonio Lei and David Loeffler. I’ll also explain recent work with Loeffler and Guido Kings where we prove an explicit reciprocity law for this Euler system, and use this to prove cases of the BSD conjecture and the finiteness of Tate–Shafarevich groups.