SOUTHERN CALIFORNIA NUMBER THEORY DAY, OCTOBER 1, 2016

ABSTRACTS

Jeff Achter, *Local densities compute isogeny classes.*

Consider an ordinary isogeny class of elliptic curves over a finite, prime field. Inspired by a random matrix heuristic (which is so strong it's false), Gekeler defines a local factor for each rational prime. Using the analytic class number formula, he shows that the associated infinite product computes the size of the isogeny class.

I'll explain a transparent proof of this formula; it turns out that this product actually computes an adelic orbital integral which visibly counts the desired cardinality. Moreover, it the new perspective allows a natural generalization to higher-dimensional abelian varieties.

This is joint work with Julia Gordon and S. Ali Altug.

Chantal David, *One-parameter families of elliptic curves with non-zero average root number.* Joint work with S. Bettin and C. Delaunay.

We investigate in this talk the average root number (i.e. sign of the functional equation) of one-parameter families of elliptic curves (i.e elliptic curves over $\mathbb{Q}(t)$, or elliptic surfaces over $\mathbb{Q}$). For most one-parameter families of elliptic curves, the average root number is predicted to be 0. Helfgott showed that under Chowla's conjecture and the square-free conjecture, the average root number is 0 unless the curve has no place of multiplicative r eduction over $\mathbb{Q}(t)$. We then build families of elliptic curves with no place of multiplicative reduction, and compute the average root number of the families. Some families have periodic root number, giving a rational average, and some other families have an average root number which is expressed as an infinite Euler product. We also show several density results for the average root number of families of elliptic curves, and exhibit some surprising examples, for example, non-isotrivial families of elliptic curves with rank r over $\mathbb{Q}(t)$ and average root number $-(-1)^r$, which were not found in previous literature.

Rachel Pries, *Reciprocity maps with restricted ramification.*

We prove a result about the Galois module structure of the Fermat curve using commutative algebra, number theory, and algebraic topology. Specifically, we extend work of Anderson about the action of the absolute Galois group of a cyclotomic field on a relative homology group of the Fermat curve. By finding explicit formulae for this action, we determine the maps between several Galois cohomology groups which arise in connection with obstructions for rational points on the generalized Jacobian. Heisenberg extensions play a key role in the result. This is joint work with R. Davis, V. Stojanoska, and K. Wickelgren.

Romyar Sharifi, *Reciprocity maps with restricted ramification.*

We will discuss two maps that naturally arise in study of the cohomology of number fields with ramification restricted to a finite set S of primes. By comparing them, we can relate the cokernel of one of them, an S-reciprocity map, to the dual Selmer groups of residual representations for newforms that satisfy congruences with Eisenstein series modulo a prime in S. This allows us to prove something of a main conjecture for these Selmer groups (and, in fact, their pseudo-cyclicity) under hypotheses that include Greenberg's conjecture.