SOLUTIONS (VERSION A)

MATH 2B – MIDTERM 1

PEYAM RYAN TABRIZIAN

Name: ______________________

Section time: (please circle)

1 – 2 PM 5 – 6 PM

Note: Please remember to fill in your name and student ID on the next page as well!

Instructions: Welcome to Midterm 1! You have 50 minutes to take this exam, for a total of 100 points. **Do not open the exam until instructed to do so.** This is a closed book and closed notes exam and calculators and/or portable electronic devices such as cell phones are **NOT** allowed and should be turned off during the entirety of the exam. Remember that you are not only graded on your final answer, but also on your work as a whole. Write in complete sentences whenever you can. If you need to continue your work on the back of the page, clearly indicate so, or else your work will be discarded. May your luck be integrable! :)

VERSION A

1. 4
2. (a) $\frac{1}{3}t^3 + \frac{1}{2}t^2 - 2t + 3$
   (b) $\pi/2$
   (c) $\sin(x^2) - \sin(e^{2x})e^x$
3. (a) $2/5\pi$
   (b) 4
4. $\pi\left(\frac{e^2}{2} + 2e - \frac{5}{2}\right)$

VERSION B

1. $\frac{1}{4}$
2. (a) $\cos(x^2) - \cos(e^{2x})e^x$
   (b) 2π
   (c) $\frac{2}{3}t^3 + t^2 - 2t + 3$
3. (a) $2/5\pi$
   (b) 8
4. $\pi\left(\frac{e^6}{2} + 4e^3 - \frac{9}{2}\right)$

Date: Friday, October 20, 2017.

Name: ______________________

Student ID: ______________________

1		20
2		30
3		30
4		20
Total		100

1. *(20 points)* Use the **definition** of the integral (in terms of Riemann sums) to evaluate

$$\int_0^2 x^3 dx$$

You are allowed to use the following facts:

$$\sum_{i=1}^{n} 1 = n, \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

PREP WORK

$$\Delta x = \frac{2-0}{N} = \frac{2}{N}$$

$$x_i = 0 + (\Delta x) i = \frac{2i}{N}$$

$$\int_0^2 x^3 \, dx = \lim_{N\to\infty} (\Delta x) \sum_{i=1}^{N} f(x_i) \qquad f(x_i) = (x_i)^3 = \left(\frac{2i}{N}\right)^3$$

$$= \lim_{N\to\infty} \frac{2}{N} \sum_{i=1}^{N} \frac{8i^3}{N^3}$$

$$= \lim_{N\to\infty} \frac{16}{N^4} \sum_{i=1}^{N} i^3$$

FACT

$$= \lim_{N\to\infty} \frac{16}{\cancel{N^4}\,N^2} \frac{\cancel{N^2}(N+1)^2}{4}$$

$$= \lim_{N\to\infty} 4 \left(\frac{N+1}{N}\right)^2$$

$$= 4$$

NOTE VERSION B : $\frac{1}{4}$

2. *(30 points total)* Find the following:

(a) *(10 points)* The position $s(t)$ of a particle with acceleration $a(t) = 2t + 1$ and $s(0) = 3$ and $v(0) = -2$.

$\Rightarrow \quad v(t) = t^2 + t + A, \quad v(0) = A = -2$

$\Rightarrow \quad v(t) = t^2 + t - 2$

$\Rightarrow \quad s(t) = \frac{1}{3}t^3 + \frac{1}{2}t^2 - 2t + B, \quad s(0) = B = 3$

$$\boxed{s(t) = \frac{1}{3}t^3 + \frac{1}{2}t^2 - 2t + 3}$$

(b) *(10 points)* $\int_{-1}^{1} (x+1)\sqrt{1-x^2}dx$

$$= \underbrace{\int_{-1}^{1} \underbrace{x\sqrt{1-x^2}}_{} dx}_{\text{ODD}} + \underbrace{\int_{-1}^{1} \sqrt{1-x^2} dx}_{\text{SEMICIRCLE}}$$

$$= 0 + \frac{\pi}{2} 1^2$$

$$= \boxed{\frac{\pi}{2}}$$

(c) *(10 points)* The derivative (with respect to x) of $\int_{e^x}^{x} \sin(t^2)\, dt$

$$\Big(\int_{e^x}^{x} \underbrace{\sin(t^2)}_{f(t)} dt \Big)' = \big(F(x) - F(e^x) \big)' \quad (F = \text{AD of } f)$$

$$= F'(x) - F'(e^x)e^x$$

$$= f(x) - f(e^x)e^x$$

$$= \boxed{\sin(x^2) - \sin(e^{2x})e^x}$$

NOTE Version B: (a) $\cos(x^2) - \cos((\ln(x))^2)\left(\frac{1}{x}\right)$ (c) $\frac{2}{3}t^3 + t^2 - 2t + 3$

(b) 2π

3. *(30 points)* Find the following:

(a) *(10 points)* The average value of the function $\cos^4(x)\sin(x)$ on $[0, \pi]$.

$$\frac{1}{\pi}\int_0^{\pi} \cos^4(x)\,\sin(x)\,dx = \frac{1}{\pi}\int_1^{-1} U^4(-dU) = \frac{1}{\pi}\left[-\frac{U^5}{5}\right]_1^{-1}$$

$$= \frac{1}{\pi}\left(\frac{1}{5}+\frac{1}{5}\right) = \frac{2}{5\pi}$$

$U = \cos(x)$, $dU = -\sin(x)\,dx \Rightarrow \sin(x)\,dx = -dU$

$U(0) = 1$, $U(\pi) = -1$

(b) *(20 points)* The area of the region enclosed by the curves

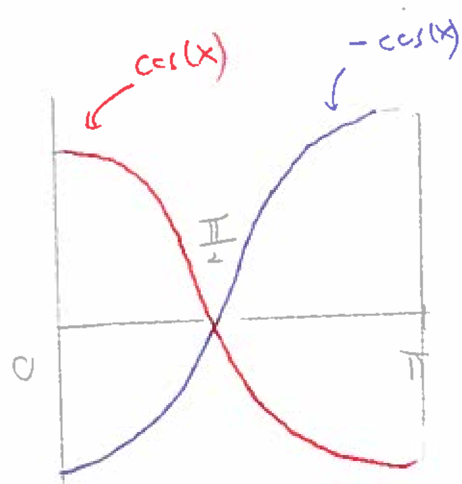
$$y = \cos(x), y = -\cos(x), x = 0, x = \pi$$

NOTE VERSION B:

(a) $\frac{2}{5\pi}$

(b) 8 (POI IS $x = \pi$)

1) PICTURE



$= 2(1-0) - 2(0-1)$

$= 4$

2) POINTS OF INTERSECTION $\cos(x) = -\cos(x) \Rightarrow 2\cos(x) = 0 \Rightarrow \cos(x) = 0 \Rightarrow x = \frac{\pi}{2}$

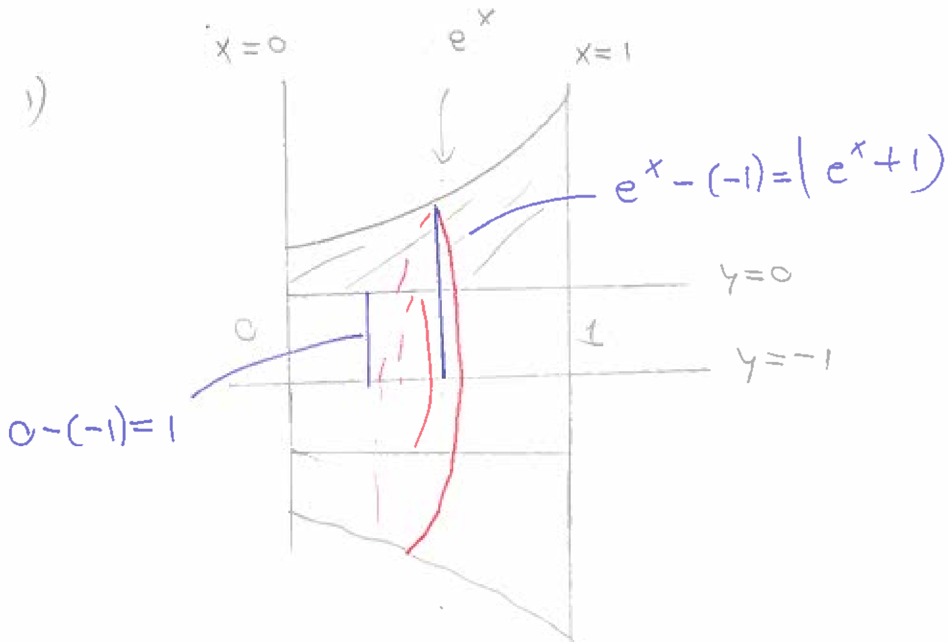
3) AREA $= \int_0^{\frac{\pi}{2}} \cos(x) - (-\cos(x))\,dx + \int_{\pi/2}^{\pi} -\cos(x) - \cos(x)\,dx$

$= 2\int_0^{\pi/2} \cos(x) - 2\int_{\pi/2}^{\pi} \cos(x) = 2\left[\sin(x)\right]_0^{\pi/2} - 2\left[\sin(x)\right]_{\pi/2}^{\pi}$

$= 2\left(\sin\left(\frac{\pi}{2}\right) - \sin(0)\right) - 2\left(\sin(\pi) - \sin\left(\frac{\pi}{2}\right)\right)$

4. *(20 points)* Find the volume of the solid obtained by rotating the region enclosed by $y = e^x$, $y = 0$, $x = 0$ and $x = 1$ about the line $y = -1$.

1)



2) WASHER METHOD

OUTER = $e^x - (-1) = e^x + 1$

INNER = $0 - (-1) = 1$

3) $$V = \pi \int_0^1 (e^x+1)^2 - 1^2 \, dx$$

$$= \pi \int_0^1 e^{2x} + 2e^x \cancel{+1} \cancel{-1} \, dx$$

$$= \pi \left[\frac{e^{2x}}{2} + 2e^x \right]_0^1$$

$$= \pi \left(\frac{e^2}{2} + 2e - \frac{1}{2} - 2 \right) = \boxed{\pi \left(\frac{e^2}{2} + 2e - \frac{5}{2} \right)}$$

NOTE VERSION B:

$$\pi \int_0^3 (e^x+2)^2 - 2^2 \, dx$$

$$= \pi \left(\frac{e^6}{2} + 2e^3 - \frac{5}{2} \right)$$